MODEL QUESTION FOR FIFTH SEMESTER B.Sc MATHEMTICS

(2014 Admission)

MM 1544- Vector Analysis

Time: 3 Hours Max. mark:80

SECTION- 1

All the ten questions are compulsory. They carry one mark each

1. Find the gradient of $\phi(x, y) = x + y$.
2. State Green's Theorem
3. Find the directional derivative of $f(x, y) = e^{xy}$ at (-2,0)
4. S.T div(curl F) =0
5. State the Conservation of Energy principle
6. Find the slope of the surface Z= xy in the direction of the vector u=i+j at (1,2,2)
7. When we say that a vector field is consertvative
8. Evaluate $\int_C ds$, if C is the line segment from (0,0) to (1,0)
9. State the fundamental theorem of work integrals
10. Given that **r** = xi+yj+zk . show that curl(**r**)= **0**

SECTION -2

Answer any eight from the following. Each question carries 2 marks

11) Use line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

12) Show that the divergence of the inverse square field $F(x, y, z) = \frac{c}{\|r\|^3}$ is zero

13) Evaluate $\int_C (1 + xy^2) ds$ from (0,0) to (1,2)along the li8ne segment that is represented by the parametric curve x= t , y = 2t, $0 \le t \le 1$

14) let $F(x, y) = 2xy^3 i + (1 + 3x^2 y^2) j = \nabla\phi$. Find ϕ

15) use Green's Theorem t to evaluate $\int_C x^2 y\, dx + x dy$ along the circular path joined the points (0,0), (1,0), and (1,2)

16) Use the divergence theorem to find the outward flux of the vector field F(x,y,z)= z k across the sphere $x^2 + y^2 + z^2 = a^2$

17) Find curl(curl F) for the function $F(x, y, z) = y^2 x\, i - 3xyz\, j + xy\, k$

18) If f and g are differentiable functions , show that $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$

19) Find the directional derivative of $f(x, y) = e^{-x} \cos y$ at $\left(0, \pi/4\right)$

20)Determine whether the vector field $F(x, y, z) = (x + y + z)i + (y - x - z)j + 3k$ is conservative on some open set

21) Define an " Inverse Square Field". State Gauss Law for Inverse Square Fields

22) Find the work done by the force field $F(x, y) = x^2\, i + y^2\, j$ along the parabola $y = x^2$ from (0,0) to (1,1)

SECTION- 3

Answer any six from the following. Each question carries 4 marks

23) Evaluate $\int_C \frac{-ydx + xdy}{x^2 + y^2}$, where C is a piece wise smooth closed curve oriented counter clockwise such that C encloses the origin

24) let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy-plane and suppose σ is oriented up. Find the flux of the vector field F(x,y,z) = x i+ y j+z k across σ

25) The temperature at a point (x,y,z) in a metal sheet is $T(x, y, z) = \frac{xyz}{1 + x^2 + y^2 + z^2}$.

Find the rate of change of temperature with respect to distance at (1 ,1 ,1) in the direction of the origin

26) Find the mass of a thin wire shaped in the form of a circular arc $y = \sqrt{9 - x^2}$, $0 \le x \le 3$ if the density function is $\delta(x, y) = x\sqrt{y}$

27) Show that the line integral $\int_C y \sin x\, dx - \cos x\, dy$ is independent of the path and hence evaluate $\int_{(0,1)}^{(\pi,-1)} y \sin x\, dx - \cos y\, dy$

28) Using Green's theorem find the work done by the force field
$F(x, y) = (e^x - y^3)i + (\cos y + x^3)j$

29) Suppose that a curved lamina σ with constant density $\delta(x, y, z) = 1$ is the portion of the paraboloid $z = x^2 + y^2$ below thw plane z =1. Find the mass of the lamina

30) Use divergence theorem to find the outward flux of the vector field
$F(x, y, z) = 2xi + 3yj + z^2k$ across the unit cube bounded by the planes x = 0, x =1, y = 0, y = 1, z = 0, and z = 1.

31) Using Stoke's Theorem , evaluate $\int_C F.dr$, where $F(x, y, z) = z^2i + 2xj - y^3k$, C is the circle $x^2 + y^2 = 1$ in the xy – plane with counter clock-wise orientation looking down the z – axis.

SECTION -4

Answer any two from the following. Each question carries 15 marks

32) Prove that a) $div(\varphi F) = \varphi divF + \nabla\phi.F$
b) $curl(\varphi F) = \varphi curlF + \nabla\phi \times F$

33) Verify divergence theorem for the function F(x,y,z) = x i+ y j+ z k , where σ is the spherical surface $x^2 + y^2 + z^2 = 1$

34)
a) Let $T(x, y) = 10 - 8x^2 - 2y^2$. Find the maximum value of a directional derivative at (2,3). Also find the unit vector in the direction in which the maximum value occurs.

b) A heat seeking particle is located at the point (2,3) in a flat metal plate whose temperature at a point (x, y) is $T(x, y) = 10 - 8x^2 - 2y^2$. Find an equation of the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

35) Verify Stoke's Theorem for the vector field $F(x, y, z) = 2zi + 2xj + 5yk$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with up ward orientation , and C to be positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy plane.