UNIVERSITY OF KERALA

MODEL QUESTION PAPER

FIRST DEGREE PROGRAMME UNDER CBCSS

SEMESTER V- MATHEMATICS

2014 admission

REAL ANALYSIS - 1

MM 1541

TIME 3hrs maximum marks 80

Section 1

All the first 10 questions are compulsory. They carry 1 mark each

1) Find all real numbers x that satisfy the inequality $\frac{1}{x} < x$

2) If S = $\left\{1 - -1^{n}/n\right\}$, find inf S

3) State supremum property of R

4) What is a Fibonacci sequence

5) Define a Cauchy sequence

6) Give an example of a bounded sequence that is not a Cauchy sequence

7) If $I_n = [0, 1/n]$ for n$\in$ N, find $\bigcap_1^{\infty} I_n$

8) Define cluster point of a set

9) What is $\lim_{x\to 0} x^2/|x|$

10) Give an example of two divergent sequences X and Y such that their sum converges

Section 2

Answer any eight questions from this section .Each question carries two marks

11) Show that there exists no rational number r whose square is 2

12) State and prove Bernoulli's inequality

13) Establish triangle inequality

14) State the order properties of R

15) State and prove the Archimedean property

16) Show that a convergent sequence of real numbers is bounded

17) Give an example of an unbounded sequence that has a convergent subsequence

18) Show that a Cauchy sequence of real numbers is bounded

19) Show that lim $\frac{\sin n}{n}$ = 0

20) Evaluate $\lim_{x \to 0} \frac{\cos x - 1}{x}$

21) Show that $\sum_{1}^{\infty} 1/\sqrt{n+1}$ is divergent

22) Show that $\sum_{0}^{\infty} \frac{1}{(n+1)(n+2)}$ =1

Section 3

Answer any six questions from this section. Each question carries 4 marks

23) State and prove density theorem

24) State and prove squeeze theorem

25) Show that the sequence $\left(1+(-1)^{n}\right)$ is not a Cauchy sequence but (1/n) is a Cauchy sequence

26) Show that $\sum 1/n^{p}$ converges if p >1 and diverges if p $\leq$1

27) Show that $\sum \frac{\cos n}{n^{2}}$ is convergent

28) Define a contractive sequence and show that every contractive sequence is a Cauchy sequence

29) Show that $\lim_{x \to 0} \sin(\frac{1}{x})$ does not exist in R

30) Prove that $\lim_{x \to 0} \cos(\frac{1}{x})$ does not exist but $\lim_{x \to 0} x\cos(\frac{1}{x})$ = 0

31) Discuss the convergence of the series whose n^{th} term is $\frac{n^2 - n}{n(n^2+2)}$

Section 4

Answer any two questions from this section. Each question carries 15 marks

32 a) State and prove nested interval property of real numbers

b) If S is a subset of R that contains at least two points and has the property that if x,y $\in$ S and x< y then[x,y] $\subset$ S ,prove that S is an interval

33 a) Let X= (x_n) and Y = (y_n) be sequences of real numbers which converges to x and y respectively and c $\in$ R. Show that the sequences X+Y, X-Y, XY, cX converges to x+y, x-y, xy and cx respectively

34 a) State and prove monotone convergence theorem

b) State and prove Bolzano Weierstrass theorem

35 a) Establish Cauchy criteria for convergence of sequence of real numbers

b) Establish sequential criteria for limit of a function