UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme
Semester VI Core Course
MM: 1643 Complex Analysis II

Time: 3 Hours **Maximum Marks: 80**

Section I

All the first 10 questions are compulsory. Each carries 1 mark.

1. Write the power series representation of $f(z) = \dfrac{1}{z-1}$ in a disc of radius 1 centered at $z = 2$.
2. If f is analytic in a disc $B(\alpha, r)$ and $f(z) = \sum_{k=0}^{\infty} c_k z^k$. What is the value of c_k?
3. What are the singularities of $\dfrac{1}{\sin \frac{\pi}{z}}$?
4. Define a pole of order m for a function $f(z)$.
5. What type of singularity the function $e^{\frac{1}{z}}$ has at $z = 0$.
6. Define removable singularity.
7. Find the residue at $z = 0$ for the function $\dfrac{1}{z + z^2}$.
8. Find the principal value of $\int_{-\infty}^{\infty} x\, dx$.
9. What is the order of the zero of $z(e^z - 1)$.
10. What type of singularity the function $\text{cosec} z$ has at $z = 0$.

Section II

Answer any 8 questions from this section.
Each question carries 2 marks

11. Find a power series representing the function $f(z) = \dfrac{1}{z^2}$, near $z = 3$. Also find the radius of convergence.
12. Show that $\int_C \exp(\frac{1}{z^2})\, dz = 0$, where C is positively oriented circle $|z| = 1$.
13. Determine and identify the singularities of $\dfrac{z^2}{1+z}$.
14. Find the residue of the function $f(z) = \dfrac{z^3 + z^2}{(z-i)^3}$, at $z = 1$.
15. Determine the order of the pole and residue at $z = 0$ for $\dfrac{\sinh z}{z^4}$.

16. Write the principal part of the function $f(z) = z\exp(\frac{1}{z})$ at its isolated singular point and determine the value of the singularity.

17. If z_0 is a pole of the function f, show that $\lim_{z \to z_0} f(z) = \infty$.

18. State jordan's lemma.

19. Find the residue of $f(z) = \cot z$ at each of its singular points.

20. Let C denote the positively oriented circle $|z| = 2$. Evaluate $\int_C \tan z\, dz$.

21. Find $\sum_1^\infty \frac{1}{n^2}$.

22. Evaluate $\sum_{k=0}^\infty (\frac{n}{k})^2$.

Section III

Answer any 6 questions from this section.

Each question carries 4 marks.

23. State and prove Cauchy's integral formula.

24. Evaluate $\int_C \frac{dz}{z(z-2)^4}$, where C is the positively oriented circle $|z-2| = 1$.

25. State and prove Cauchy's residue theorem.

26. Describe three types of isolated singular points.

27. Find the value of the integral $\int_C \frac{dz}{z^3(z+4)}$ taken counter-clockwise around $|z| = 2$.

28. If z_0 is a removable singular point of a finite function f, show that f is analytic and bounded in some neighbourhood of $0 < |z - z_0| < \epsilon$ of z_0.

29. Show that $\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}$ $(-1 < a < 1)$.

30. Evaluate $\sum_1^\infty \frac{1}{n^2+1}$.

31. Find $\sum_0^\infty \binom{2n}{n} \frac{1}{5^n}$.

Section IV

Answer any 2 questions from this section.

Each question carries 15 marks.

32. **(a)** Use cauchy's residue theorem to evaluate $\int_C \frac{z+1}{z^2-2z}$ around the circle $|z| = 3$ in the positive sense.

(b) Show that $\operatorname*{Res}_{z=\pi i} \dfrac{z - \sinh z}{z^2 \sinh z} = \dfrac{i}{\pi}$

33. **(a)** State and prove Casorati-Weierstrass theorem.

(b) Evaluate $\displaystyle\int_0^\infty \frac{x^2}{1+x^6}\, dx.$

34. **(a)** Show that $\displaystyle\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2} = \frac{2\pi}{e^3}.$

(b) Show that $\displaystyle\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}.$

35. **(a)** Use residues to evaluate $\displaystyle\int_0^\pi \frac{d\theta}{5+4\cos\theta}$.

(b) Use residues to find the Cauchy principal value of $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x\, dx}{x^2+4x+5}.$