KERALA UNIVERSITY

Model Question Paper- M. Sc. Examination 2020 admission onwards

Branch : Mathematics

MM 231 COMPLEX ANALYSIS

Time: 3 hours Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8

Each question carries 3 marks

1. Define bounded variation of a complex function f and give an example of a function which is of bounded variation.Also define total variation.
2. Define absolute convergence of a series. Show that absolute convergence implies convergence.
3. Define an entire function and give an example.Alsodefine index of a curve γ with respect to a point a.
4. Show that FEP homotopic is an equivalence relation.
5. Define different types of isolated singularities and give examples for each.
6. Define maximum principle an state first version of maximum Modulus theorem.
7. Evaluae $\frac{1}{2\pi i}\int\limits_{\gamma}\frac{2z+1}{z^2+z+1}dz$, where γ is the circle $|z|=4$
8. Define Mobius transformation. Also show that a Mobius transformation has atmost two fixed pionts. . $5\times 3=15$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9. .

A. .

a. If $\sum a_n(z-a0^n$ is a given power series with radius of convergence R, then prove that $r=\lim|\frac{a_n}{a_{n+1}}|$ if the limit exists.

b. Find the radius of convergence of $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}z^{n(n+1)}$

OR

B. *a*. Dfine a smooth function. If $\gamma:[a,b]\to\mathbb{C}$ is piecewise smooth, prove that γ is of bounded variation and $v(\gamma)=\int\limits_a^b|\gamma'(t0|dt$.

b. Find $\int_\gamma z^{-\frac{1}{2}} dz$ where γ is the upper half of the unit circle from 1 to -1.

10. .

A. *a.* Let $\phi : [a, b] \times [c, d] \to \mathbb{C}$ be a continuous function and define $g : [c, d] \to \mathbb{C}$ by $g(t) = \int_a^b \phi(s, t) ds$. Prove that g is continuous. Also show that if $\frac{\partial \phi}{\partial t}$ exists and is continuous on $[a, b] \times [c, d]$ then g is continuously differentiable and $g'(t) = \int_a^b \frac{\partial}{\partial t} \phi(s, t) ds$

OR

B. *a.* State ans prove Liouvilli's theorem.

b. Let f and g be two analytic functions on a region G. Prove that $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ has alimit point in G

11. .

A. *a.* Let γ be a rectifiable curve and suppose π is a function defined and continuous on $\{\gamma\}$ and for each $m \geq 1$, let $F_m(z) = \int_\gamma \phi(w)(w - z)^{-m} dw$ for each $z \notin \{\gamma\}$. Prove that each F_m is analytic on $\mathbb{C} - \{\gamma\}$ and $F'_m(z) = mF_{m+1}(z)$.

b. Find $\int_\gamma \frac{z^2 + 1}{z^2 - 1} dz$ where γ is the circle $|z - 1| = 1$

OR

B. *a.* State and prove Morera's theorem.

b. Let G be an open set which is a-star shaped. If γ_0 is the curve which is constantly equal to a then prove that every closed rectifiable curve in G is homotopic to γ_0

12. .

A. *a.* Derive Laurent series development of an analytic function in the $ann(a, R_1, R_2)$

OR

B. *a.* State and prove Casorati-Weirstrass theorem.

b. State and prove Rouche's theorem.

13. .

A. *a.* State and prove Schwarz's lemma.

b. State and prove Maximum modulus theorem third version.

OR

B. *a.* Prove that a mobius transformation takes circles to circles

b. State and prove Orientation principle.

. $5 \times 12 = 60$