KERALA UNIVERSITY

Model Question Paper- Third Semester M. Sc. Examination
Branch : Mathematics
MM 233 : Elective I : OPERATIONS RESEARCH

Time: 3 hours | Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8
Each question carries 3 marks

1. What do you mean by standard form of an LPP?

2. Explain the procedure of solving a LPP using graphical method.

3. Using Vogel's approximation method, find an initial basic feasible solution to the transportation problem :

	M_1	M_2	M_3	M_4	Supply
W_1	2	2	2	1	3
W_2	10	8	5	4	7
W_3	7	6	6	8	5
Demand	4	3	4	4	

4. Write a note on assignment problem. Write the mathematical model of assignment problem.

5. Define the terms 'most probable time', 'optimistic estimate' and 'pessimistic estimate' in connection with project network.

6. What do you mean by non-linear programming problem? Define Lagrangian function for the non-linear programming problem : Minimize $f(X)$ subject to $g_i(X) \leq 0$, $i = 1, 2, \ldots, n$.

7. Prove that if $F(X, Y)$ has a saddle point (X_0, Y_0) for all $Y \geq 0$, then $G(X_0) \leq 0$, $Y_0' G(X_0) = 0$.

8. Explain the computational economy in dynamic programming. $5 \times 3 = 15$

Part B

Anwer all questions from 9 to 13
Each question carries 12 marks

9. A. Solve the following LPP by simplex method:

$$\begin{aligned} \text{Maximize } Z = 5x_1 + 2x_2 &+ 3x_3 - x_4 + x_5 \\ \text{Subject to} \quad x_1 + 2x_2 + 2x_3 + x_4 &= 8 \\ 3x_1 + 4x_2 + x_3 + x_5 &= 7 \\ x_1,\, x_2,\, x_3,\, x_4,\, x_5 &\geq 0 \end{aligned}$$

OR

B. Solve the following LPP by Big M Method:

$$\begin{aligned} \text{Maximize } Z = -3x_1 + x_2 &+ x_3 \\ \text{Subject to} \quad x_1 - 2x_2 + x_3 &\leq 11 \\ -4x_1 + x_2 + 2x_3 &\geq 3 \\ 2x_1 - x_3 &= -1 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

10. A. Find a solution to the following transportation problem :

	D_1	D_2	D_3	D_4	Supply
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	

(Start with an initial basic feasible solution by North West corner rule).

OR

B. Solve the assignment problem:

	J_1	J_2	J_3	J_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

11. A. Consider a project with 5 jobs A, B, C, D and E with the following job sequence: Job A precedes C and D; Jobs B precedes D; Job C and D precede E. The completion times for A, B, C, D and E are 3, 1, 4, 2 and 5 respectively. Construct the project network, find earliest time, latest time and slack time of each event.

OR

B. Consider a project consisting of nine jobs (A, B, $\ldots I$) with the following precedence relations and time estimates:

Job	Predecessor	Optimisstic Time (a)	Most Probable Time (m)	Pessimistic Time (b)
A	–	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B, C	1	4	7
E	A	8	8	8
F	D, E	5	14	17
G	C	3	12	21
H	F, G	3	6	9
I	H	5	8	11

a) Draw the project network for the above problem

b) Determine the expected duration and variance of each job.

c) What is the expected length of the project and its variance.

12. A. Find the minimum of

$$
\begin{aligned}
f(X) = (x_1+1)^2 &+ (x_2-2)^2 \\
\text{subject to } g_1(x) = x_1 - 2 &\leq 0, \\
g_2(x) = x_2 - 1 &\leq 0, \\
x_1,\ x_2 &\geq 0.
\end{aligned}
$$

OR

B.

$$
\begin{aligned}
\text{Minimize } f(X) &= -x_1 - x_2 - x_3 + \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \\
\text{subject to } g_1(X) = x_1 + x_2 + x_3 - 1 &\leq 0 \\
g_2(X) = 4x_1 + 2x_2 - \frac{7}{3} &\leq 0 \\
x_1,\ x_2,\ x_3 &\geq 0.
\end{aligned}
$$

13. A. a) Prove that in a serial two-stage minimization or maximization problem if

(i) the objective function ϕ_2 is a separable function of stage returns $f_1(X_1, U_1)$ and $f_2(X_2, U_2)$, and

(ii) ϕ_2 is monotonic nondecreasing function of f_1 for every feasible value of f_2,

then the problem is decomposable.

b) Minimize $u_1^2 + u_2^2 + u_3^2$ subject to $u_1 + u_2 + u_3 \geq 10$, $u_1,\ u_2,\ u_3 \geq 0$.

OR

B. a) Write an algorithm to find the shortest path in a minimum path problem.

b) Determine the maximum of $u_1 u_2 u_3$ subject to $u_1 + u_2 + u_3 = 5$, $u_1,\ u_2,\ u_3 \geq 0$.

. $5 \times 12 = 60$