University of Kerala

First degree programme in Mathematics

Fifth Semester B.Sc. Degree model questions

COMPLEX ANALYSIS

MM 1542

(2014 Admissions onwards)

Time: 3 Hours Max. Marks: 80

All the **first 10** questions are **compulsory.** They carry **1** mark **each** .

1. Express $\frac{(4-i)(3+i)}{(2-i)}$ in the form $a+ib$
2. Find the square root of $4i$
3. Show that $|z|^2 = z\, z^{-}$.
4. Find the radius of convergence of the series $\sum_{n=0}^{\infty} z^{n^2}$
5. If $z = x + iy$, find $|e^z|$
6. State rectangle theorem.
7. Express $3+2i$ in polar form .
8. Write an example for a Cauchy sequence in Complex plane .
9. Let $\{z : z = z^{-}\}$. What is it geometrically means ?
10. Write the power series representation of e^z.

(10x1=10 Marks)

Answer **any 8** questions from among the questions **11 to 22 .** These questions carry **2** marks **each.**

11. Find the sum of complex numbers $3 + 2i$ and $1 + i$ geometrically.

12. Find the cube roots of unity.

13. Prove that $\{z_n\}$ converges if and only if $\{z_n\}$ is a Cauchy Sequence .

14. Using Cauchy Riemann equation, verify $-x^2 + y^2 - 2xy\, i$ is analytic.

15. Find the radius of convergence of. Is the series $\Sigma(1/2)^n$ converge or diverge.

Justify your answer.

16. Prove that f is constant if $f = u + iv$ is analytic in a region D and u is Constant.

17. Let $f(z) = \frac{1}{z}$ and $C: z(t) = R\, cost + i\, R\, sint, 0 \leq t \leq 2\pi, R \neq 0$. Then

Find $\int_c f(z)dz$.

18. Evaluate $\int_c f(z)dz$ where $f(z) = x^2 + y^2$ and $C\ is\ z(t) = t^2 + i\, t^2$

$0 \leq t \leq 1.$

19. Solve $x^3 + 4x + 2$ by Cubic Method.

20. Is the polynomial $x^2 + y^2 - 2xiy$ is analytic. Justify your answer.

21. Suppose $C\ is\ given\ by\ z(t)\ , a \leq t \leq b$. Then prove that

$\int_c f = \int_c f$.

22. Let $f(z) = |z|^2$. Is f differentiable at $z = 0$. Justify.

(8x2=16 Marks)

Answer **any 6** questions from the questions **23** to **31**.These questions carry **4** marks **each**.

23. Geometrically represent the following sets.

a) $\{z : -\theta < arg\ z < \theta\}$.

b) $\{z : \left|Arg\ z - {}^{\pi}/_{2}\right| < {}^{\pi}/_{2}\}$.

24. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$ and interpret the Result geometrically.

25. Prove that if a polynomial $P(x, y)$ is analytic if and only if $P_x = iP_y$.

26. State and prove uniqueness theorem for power series.

27. Suppose f is derivative of an analytic function F - that is $f(z) = F'(z)$ Where F is analytic on the smooth curve C.

Then $\int_c f(z)dz = F\big(z(b)\big) - F(z(a))$.

28. a) Evaluate $\int_c (z - i)dz$ where c is the parabolic segment $z(t) = t + i\,t^2$ $-1 \le t \le 1$.

b) Also find the above integral along the straight line $-1 + i$ to $1 + i$.

29. Let C be a smooth curve ; let f and g be continuous function on C; and Let α be any complex number. Then

a) $\int_c (f(z) + g(z))dz = \int_c (f(z)dz + \int_c g(z))dz$.

b) $\int_c \alpha f(z)dz = \alpha \int_c (f(z)$.

30. Verify the following identities

a) $\sin 2z = 2 \sin z \cos z$.

b) $sin^2 z + cos^2 z = 1$.

31. Is the following polynomials are analytic. Verify

a) $P(x, y) = x^3 - 3xy^2 - x + i\,(3x^2 - y^3 - y)$

b) $P(x, y) = 2xy + i\,(y^2 - x^2)$. (6x4=24 marks)

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Suppose that $f(z) = \sum_{n=0}^{\infty} c_n z^n$ converges for $|z| < R$. Then $f(z)'$ exists

And equals $\sum_{n=0}^{\infty} nc_n z^{n-1}$ throughout $|z| < R$.

b) The power series are infinitely differentiable within their domain of

Convergence

33. a) If $f = u + iv$ is differentiable at Z, f_x and f_y exists there and satisfy the

Cauchy Riemann equation $f_y = if_x$.

b) Is the converse of the above statement is true. Justify your answer.

c) Show that $f(z) = Rez$ is nowhere differentiable.

34. a) Suppose f is entire and Γ is the boundary of a rectangle R.

Then $\int_{\Gamma} f(z)dz = 0$.

b) State and prove integral theorem.

35. a) Suppose $G(t)$is a continuous complex- valued function of t . Then

$\int_a^b G(t)dt \ll \int_a^b |G(t)|dt.$

b) Suppose that C is a smooth curve of length L, that f is continuous on C,

and that $f \ll M$ throughout C. Then $\int_C f(z) \ll ML.$

(15x2=30)