UNIVERSITY OF KERALA

MODEL QUESTION PAPER

FIRST DEGREE PROGRAMME UNDER CBCSS

SEMESTER V- MATHEMATICS

2014 admission

Abstract Algebra 1

MM 1545

Part A

All the first 10 questions are compulsory. They carry 1 mark each

1) If * is any binary operation on a set S, then a $*$ a = a for all a $\in$ S .Write true or false
2) Is the binary operation * defined on Q by a $*$ b = ab/2 associative
3) When we say that two algebraic binary structures to be isomorphic
4) What are the generators of Z_4
5) Define a cyclic group
6) Every abelian group is cyclic .Write true or false
7) Compute (1,4,5)(7,8)(2,5,7)
8) Every group of prime order is abelian. Write true or false
9) Determine whether the binary operation defined on Z by a $*$ b = ab gives a group structure of Z
10) Define the terms cycle and length of a cycle

Part B

Answer any eight questions from this section .Each question carries two marks

11) Show that (2Z,+) is isomorphic to (Z, +)
12) Prove that a binary structure (S .*) has at most one identity element
13) Show that the left and right cancellation law holds in a group
14) Find the order of the cyclic subgroup of Z_4 generated by 3
15) Show that every cyclic group is abelian
16) Find the number of elements in the cyclic subgroup of Z_{30} generated by 25
17) Compute $\tau^2\sigma$ where $\sigma = \begin{pmatrix} 1,2,3,4,5,6 \\ 3,1,4,5,6,2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1,2,3,4,5,6, \\ 2,4,1,3,6,5 \end{pmatrix}$
18) Find the number of elements in the set $\{\sigma \in S_4 : \sigma(3) = 3\}$
19) Find the orbits of the permutation $\begin{pmatrix} 1,2,3,4,5,6,7,8 \\ 3,8,6,7,4,1,5,2 \end{pmatrix}$ in S_8

20) Find the partition of Z_6 into cosets of the subgroup H = {0, 3 }

21) Find all cosets of the subgroup $\langle 4 \rangle$ of Z_{12}

22) Find the index of $\langle 3 \rangle$ in the group Z_{24}

Part C

Answer any six questions . Each question carries 4 marks

23) Show that Q^+ with * defined by a*b = ab/2 is a group

24) Prove that the identity element and inverse of each element in a group are unique

25) Describe all the elements in the cyclic subgroup of GL(2,R) generated by $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

26) Prove that the subgroup of a cyclic group is cyclic

27) Prove that every permutation σ of a finite set is a product of disjoint cycles

28) Let σ = (1,2,5,4)(2,3) in S_5 .Find the index of $\langle \sigma \rangle$ in S_5

29) Exhibit the left cosets and the right cosets of the subgroup 3Z of Z

30) Find the order of (8,4,10) in the group $Z_{12} \times Z_{60} \times Z_{24}$

31) Express $\begin{pmatrix} 1,2,3,4,5,6,7,8 \\ 8,2,6,3,7,4,5,1 \end{pmatrix}$ as a product of disjoint cycles and as a product of transpositions

Part D

Answer any two questions from this part .Each question carries 15 marks

32) a) Let G be a group and a $\in$ G. Show that H= { a^n : n $\in$ Z } is a subgroup of G and is the smallest subgroup of G that contains a

b) Write the order of the cyclic subgroup of U_5 generated by cos4π/5 +i sin4π/5

c) Write the group Z_6 of 6 elements. Compute the subgroups $\langle 0 \rangle$ and $\langle 1 \rangle$

33) a) Let G be a cyclic group with generator a . If the order of G is infinite prove that G is isomorphic to $\langle Z, + \rangle$ .If G has finite order n prove that G is isomorphic to $\langle Z_n, +_n \rangle$

b) Find the number elements in the cyclic subgroup of Z_{42} generated by 30

34) a) State and prove Cayley's theorem.

b)Write the group S_3. Find the cyclic subgroups $\langle \rho_1 \rangle$, $\langle \rho_2 \rangle$, $\langle \mu_1 \rangle$ of S_3

35) a) Let H be a subgroup of a finite group G. Prove that order of H is a divisor of order of G

b) Prove that every group of prime order is cyclic

c) Let A be a nonempty set and S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication.