MODEL QUESTION PAPER-2015

13.501 ENGINEERING MATHEMATICS-IV (E)

(PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES)

Time: 3 Hours
Max. **Marks**: 100

PART-A

Answer all questions. Each question carries 4 marks

1. Explain (i) bisection method (ii) Euler method

2. Evaluate by $\int_1^2 \frac{dx}{x}$ Simpson's on third rule taking $h = \frac{1}{4}$

3. Let X be a random variable with probability density function $f(x) = \frac{10}{x^2}$ for $x > 10$ and $f(x) = 0$ for $x \le 10$. i) Find $P[X > 20]$ (ii) What is the cumulative distribution of X?

4. If X is a Poisson random variable such that $P[X = 2] = 9P[X = 4] + 90P[X = 6]$ .Find its mean and variance

5.If the joint p.d.f of the random variables X and Y is $f(x, y) = 6e^{-2x-3y}$ $, x \ge 0, y \ge 0$,find the marginal density and conditional density of Y given X

PART-B

Answer any one full question from each module. Each full question carries 20 marks

Module- I

6 a. Find a real root of the equation $x^3 - 2x - 5 = 0$ by Regula -falsi method.

b. Find a real root of the equation $3x = \cos(x) + 1$ by Newton-Raphson method.

c. Using Newton's forward interpolation formula find the value of f(1.6) given

x:	1	1.4	1.8	2.2
f(x):	3.49	4.82	5.96	6.5

7a.Apply Gauss elimination method to solve x+4y-z=-5; x+y-6z=-12; 3x-y-z = 4

a. Apply Gauss-Seidel iteration method to solve the equations

20x+y-2z=17; 3x+20y-z=-18; 2x-3y+20z=25

b. Apply Lagrange's interpolation formula to find the value of y when x=10, if the following values of x and yare given.

x:	5	6	9	11
f(x):	12	13	14	16

Module-II

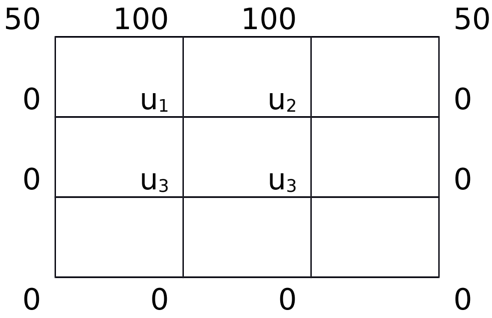
8 a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule.

b. Find by Taylor series method the value of y at x=0.1 to five places of decimals from $\frac{dy}{dx} = x^2 y - 1 \quad , y(0) = 1$

c. Using modified Euler method ,find an approximate value of y when x=0.3, given that $\frac{dy}{dx} = x + y,$ and y=1 where x=0

9 a. Apply Runge- Kutta fourth order method to find an approximate value of y for x=0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2,$ given that y=1 where x=0

b. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh given below with the boundary values as shown in figure



Module-III

10 a. The density function of a Random variable X is given by $f(x)=kx(2-x)$ for $0 \le x \le 2$ Find k, mean and variance

b. The failure pattern of an electronic system is known to follow an exponential distribution with mean time to failure of 500 hrs. Find the probability that the system failure occurs within 300 hrs.

c. In a normal distribution , 31% of the items are under 45 and 8% are over64.Find the mean and standard deviation of the distribution

11 a. Five fair coins are flipped and the number of heads (X) are counted. If the outcomes are assumed to be independent, find i, $P[X \ge 3]$ ii, Find the mean and variance of X.

b. The weekly wages of 1000 workers are normally distributed around a mean of Rs.500 with a standard deviation of Rs.50. Estimate the number of workers, whose weekly wages will be

(i) between Rs.400 and Rs.600

(II) less than Rs.400

(iii) more than Rs.600

c. Buses arrive at a specified stop at 15-minute intervals starting at 7a.m.If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7.30,find the probability that he waits

i) less than 5 minutes for a bus

ii) more than 10 minutes for a bus

Module IV

12a.Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary(WSS) if A and θ is uniformly distributed random variable in $(0, 2\pi)$

b. A stationary process has an auto correlation given by $R_{XX}(\tau) = 2 + 6e^{-3|\tau|}$ .Find the mean and variance of the process.

c. Suppose that the customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute; find the probability that during the interval of 2 minutes (i)exactly 4 customers arrive and (ii)more than 4 customers arrive.

13.a Consider the Random process $X(t) = \cos(t + \phi)$ where ϕ is a random variable with density function $f(\phi) = \frac{1}{\pi} \quad for \; \frac{-\pi}{2} < \phi < \frac{\pi}{2}$. Check whether or not the process stationary

b. If $\{X(t)\}$ and $\{Y(t)\}$ are independent wide sense stationary processes with zero means ,find the autocorrelation function of $\{Z(t)\}$ when $z(t) = a + bX(t) + cY(t)$

c. Find the mean and variance of the Poisson process.

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