**Model Question Paper**

## Second Semester MSc. Degree Examination

# Statistics with Specialization in Data Analytics

**(2020 Admission onwards)**

## STSD 221: DISTRIBUTION THEORY

Time 3 hours　　　　　　　　　　　　　　　　　　　　Max: 75 marks

**Part- A**

(Answer any **FIVE** questions, each carries **3** marks)

1. Define moment generating function(mgf). Derive the mgf of Binomial distribution.
2. Find the mean and variance of the lognormal distribution.
3. Define bivariate normal distribution.
4. Define mixture distributions.
5. If X is a continuous variable with pdf f(x) find the pdf of Y=|X|.
6. If X~U(0,1) distribution, find the distribution of $Y= -\log_e X$
7. Define Order Statistics. Give an example.
8. If $X_1, X_2, \ldots, X_n$ are iid random variables with pdf f(x)= 1 if 0<x<1; 0 otherwise, then write the pdf of the $r^{th}$ order statistic.
9. Distinguish between statistic and parameter. Give an example for each.
10. Give any two applications of t and F-distributions each.

**Part-B**

(Answer any **THREE** questions, each carries **12** marks)

11. State and prove the lack of memory property of geometric distribution
12. Define hyper geometric distribution. Give an example of a case where the distribution arise. Derive mgf and hence find its mean and variance.

13. a) Show that $E^2(XY) \leq E(X^2).E(Y^2)$
    b) If $E(X^2)<\infty$, prove that V(X)=V(E(X/Y))+E(V(X/Y))

14. Let X and Y be jointly distributed with pdf

$$f(x,y)= \frac{1+xy}{4} \quad \text{where } |x|<1, |y|<1; \ 0, \text{ otherwise}$$

Show that X and Y are independent.

15. a) If X and Y are independent U(0,1) random variables, find the distribution of X-Y
    b) If (X,Y) follows bivariate normal distribution, then show sthat non-correlation between X and Y imply independence and vice-versa.

16. Let X and Y be independent $G(\alpha_1,\beta)$ and $G(\alpha_2,\beta)$ respectively. Prove that $\frac{X}{X+Y}$ ~Beta distribution with parameters $\alpha_1$ and $\alpha_2$.

## Part-C

(Answer any **TWO** questions, each carries **12** marks)

17. Derive the joint distribution of the $r^{th}$ and $s^{th}$ order statistics of a random sample of size n drawn from a population with distribution function F and probability density function f.
18. Derive the distribution of the sample median based on a sample from an exponential distribution.
19. a) If t is distributed according to student t distribution with n degrees of freedom, then shows that $t^2$ is distributed as F(1,n).
    b) Define non-central Chi square distribution. Write down the pdf and indicate the non-centrality parameter.
20. If $\overline{X}$ is the sample mean and $S^2$ is the sample variance of a random sample from a normal population, show that $\overline{X}$ and $S^2$ are independent.

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