KERALA UNIVERSITY
Model Question Paper- M. Sc. Examination2021 admission onwards
Branch : Mathematics
MM 233- Algebraic Topology

Time: 3 hours | Max. Marks:75

Part A
Answer any 5 questions from among the questions 1 to 8
Each question carries 3 marks

1. Define an *n-pseudomanifold* and give an example?
2. Give/Draw two examples for properly joined simplexes?
3. Give/Draw two triangulations of the Möbius strip?
4. Define the *p-dimensional* homology group $H_p(K)$? What are the homology groups for the n-sphere, $S^n, n \geq 1$?
5. Give an example of a simplicial mapping?
6. Give an orientation for the complex $Cl(\sigma^2)$ and find the first barycentric subdivision?
7. Find the fundamental group $\pi_1(\mathbb{R}^2 \backslash \{p\})$?
8. Give an example of a *covering projection*? . $5 \times 3 = 15$

Part B
Answer all questions from 9 to 13
Each question carries 12 marks

9. A. i. Let K be an oriented complex, σ^{p+1} an oriented $p+1$-simplex of K and σ^{p-1} a $p-1$-face of σ^p. Show that

$$\sum_{\sigma^p \in K} [\sigma^{p+1}, \sigma^p][\sigma^p, \sigma^{p-1}] = 0?$$

ii. Prove that set of $k+1$-points in $\mathbb{R}^n$ is geometrically independent if and only if $p+1$ of the points lie in a hyperplane of dimension less than or equal to $p-1$?

OR

B. i. Prove that a set $A = \{a_0, a_1, \ldots, a_k\}$ of points in $\mathbb{R}^n$ is geometrically independent if and only if the set of vectors $\{a_1 - a_0, \ldots, a_k - a_0\}$ is linearly independent?

ii. Let K denote the closure of a 3-simplex $\sigma^2 = \langle a_0 a_1 a_2 \rangle$ with vertices ordered by $a_0 < a_1 < a_2$. Use this given order to induce an orientation on each simplex of K, and determine all incidence numbers associated with K?

10. A. . State and prove the Euler-Poincaré theorem?

OR

B. i. Show that there are only five regular, simple polyhedra?

ii. Let K be an oriented complex. Show that $B_p(K) \subset Z_p(K)$ for each integer p such that $0 \leq p \leq n$, where n is the dimenstion of K?

11. A. . State and prove the simplicial approximation theorem?

OR

B. i. Show that if $m \neq n$, S^m is not homeomorphic to S^n?

ii. Show that there is a vector field on S^n, $n \geq 1$ if and only if n odd?

12. A. . Show that the set $\pi_1(X, x_0)$ is a group under the '$\circ$' operation?

OR

B. i. State the *covering homotopy property*?

ii. Show that the fundamental group $\pi_1(S^1)$ is isomorphic to the group $\mathbb{Z}$ of integers under addition?

13. A. State and prove the *covering path property*?

OR

B. i. Show that there is no continuous map $f : S^n \to S^{n-1}$ for which $f(-x) = -f(x)$ for all $x \in S^n, n \geq 1$?

ii. Let $h : S^2 \to \mathbb{R}^2$ be a continuous map. Prove that there is at least one pair $x, -x$ of antipodal points for which $h(x) = h(-x)$?

$5 \times 12 = 60$