Fourth Semester B.Tech Degree Examination (2013 scheme)
13.401: Engineering Mathematics III (U)

MODEL QUESTION PAPER

Time: 3 hours MAXIMUM MARKS: 100

PART A

Answer all questions. Each question carries 4 marks

1. Prove that an analytical function with a constant argument is constant
2. Define critical point and invariant point of a transformation. Find the critical points and invariant points of $W = \frac{1}{z}$
3. Evaluate $\int_c |z| = 1$, where c is the last half of the unit circle $|z| = 1$from z = -i to z = +i
4. Using Newton Ralphson method, find the real root of $x log_{10} x = 1.2$, correct to 3 decimal places. Given that it is near to 2.
5. Using Taylor's series method, obtain the solution of $\frac{dy}{dx} = 3x + y^2$. Given that y (0) = 1. Also find the value of y for x= 0.1, correct to 4 places of decimals.

PART B

Answer 1 full question from each module. Each question carries 20 marks.

Module I

6. A) show that the function $f(z) = \frac{x^3 y(y-ix)}{x^6+y^2}$ for $z \neq 0$, even though Cauchy – Riemann equations are satisfied at z = 0

   B) Find the analytic function f(z) = u+iv, where $u + v - \frac{x}{x^2+y^2}$

   C) Discuss the transformation $w = e^z$
7. A) If f(z) is an analytic function of z, prove that $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|f(z)|^2 = 4|f^{'}(z)|^2$

   B) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find its harmonic conjugate and hence find the corresponding analytic f(z) = u+iv

   C) Find the bilinear transformation which maps the points z =1,i,-1 into the points w = i,0,-i respectively. Also find the image of $|z| < 1$, under this transformation.

Module II

8. A) Using Cauchy's integral formula evaluate $\int_c \frac{cos\pi z^2 + sin\pi z^2}{(z-1)(z-2)} dz$ where c is the circle $|z| = 3$

   B) Obtain the Laurent's series expansion of $\frac{7z-2}{z(z+1)(z+2)}$ in the region $1 < |z+1| < 3$

   C) Evaluate $\int_0^{\infty} \frac{1}{(a^2+x^2)^2} dx$

9. A) Find the nature and location of singularities of the function i) f(z) $= \frac{1}{z\sin z}$

ii) f(z) $= z^4 e^{\frac{1}{z}}$

B) Show that $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta = \frac{\pi}{6}$

C) Show that $\int_0^{\infty} \frac{x\sin x}{x^2+a^2}\, dx = \frac{\pi}{2} e^{-a}$

Module III

10. A) Find the real root of the equation $x^3 - 4x - 9 = 0$ which lies between 2 and 3, by Regula falsi method

B) Solve by Gauss-Seidel iteration method

$10x - 2y + z = 12;\ x + 9y - z = 10;\ 2x - y + 11z = 20$

C) Use Lagrange's interpolation formula to find f(1), from the following data

x:	-1	0	2	3
y:	-8	3	1	2

11. A) Using bisection method, find the root of the equation $e^x - x = 2$ lying between 1 and 1.4 correct to five places of decimals

B) The population of a town is given below

Year x:	1941	1951	1961	1971	1981	1991
Population in lakhs y:	20	24	29	36	46	51

Estimate the population increase during 1946 to 1976 by Newton's interpolation formula

C) Solve by Gauss elimination method

$x - y + 3z - 3w = 3;\ 2x + 3y + z - 11w = 1;$

$5x - 2y + 5z - 4w = 5;\ 3x + 4y - 7z + 2w = -7$

Module IV

12. A) Find the value of $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\, dx$ using

i) Trapezoidal rule

ii) Simpson's rule, by taking $h = \frac{1}{12}$

B) Apply Runge-Kutta method to find an approximate value of y when x = 0.7; Given that $\frac{dy}{dx} = y - x^2$ and y(0.6) = 1.7379

C) Using Euler's method find an approximate value of y corresponding to x = 1, Given that $\frac{dy}{dx} = x + y$ and y(0) = 1. Take h = 0.25

13. A) Solve Laplace's equation $u_{xx} + u_{yy} = 0$ satisfying the following conditions

u(0,y) = 0; u(3,y) = 8 + 2y, for $0 \le y \le 3$

u(x,0) = x^2, u(x,3) = 3 x^2, for $0 \le x \le 3$

B) Using modified Euler's method obtain the values of y(0.1) and y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, giventhat y =1 when x = 0;