Syllabus for M.Phil. (Statistics) Entrance Examination

Part-I

Research Methodology

Objectives and types of research: Motivation and objectives – Research methods vs Methodology. Types of research – Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical. Research Formulation: Defining and formulating the research problem - Selecting the problem - Necessity of defining the problem - Importance of literature review in defining a problem –Development of working hypothesis.
Research design and methods : Basic Principles and need of research design —Features of good design – Important concepts relating to research design – Developing a research plan - Exploration, Description, Diagnosis, Experimentation. Determining experimental and sample designs; Reporting and thesis writing : Structure and components of scientific reports - Types of report – Technical reports and thesis – Significance – Different steps in the preparation –Layout, structure and Language of typical reports – Illustrations and tables - Bibliography, referencing and footnotes –Oral presentation – Planning – Preparation – Practice – Making presentation – Use of visual aids - Importance of effective communication.

Part-II

Statistics

Unit 1: Additive and countably additive set functions. Counting measure-examples. Measurable functions and properties. Sequences of measurable functions. Convergence in measure, convergence in mean, convergence a.e and convergence in pth mean. Metric space-Metric in R^n, open set, closed set, limit point of a set, Cauchy sequence, Vector spaces- basis and dimensions, orthogonal basis-Gram Schmidt orthogonolization- Linear transformation, eigen values and eigen vectors, Cayley-Hamilton theorem, canonical form, diagonal form, triangular form, Jordan form, Quadratic forms, reduction of quadratic forms, derivative of a function with respect to a vector, with respect a matrix.

UNIT 2: Random variables and distribution function. Univariate discrete and continuous distributions, Bivariate distributions- Joint, conditional and marginal distributions, p.g.f. and m.g.f. of bivariate random vector, Multinomial and multivariate normal distributions, Sampling distributions and applications. Distributions of functions of random variables, Order statistics, Distributions of order statistics, Distribution of quadratic forms in normal variables. Tests of hypotheses-one sample and two sample problems, Tests of hypothesis about mean vector of a multivariate normal distribution, Hotelling's T^2 and Mahalanobi's D^2. Classification problem, classifications to one of k multivariate normal populations.

UNIT 3 : Probability space, limit of sequence of events, monotone and continuity properties of probability measure, addition theorem, independence of finite number of events, sequence of events, tail events and tail fields, Borel-Cantelli lemma, Borel zero-one law. Conditional Probability and Bayes Theorem. Stochastic convergence in distributions, convergence in probability, almost sure convergence and convergence in the rth mean, their interrelationships, examples and counter examples, Characteristic function and their elementary properties, infinite divisibility of distributions. Stochastic convergence of series of random variables, Weak and strong law of large numbers, central limit theorem Family of random variables and their distributions. Introduction to Stochastic processes, time and state space, classification of stochastic processes, processes with stationary independent increments, Markov process, renewal process, martingales. Markov chains, transition probability matrix, n-step transition probability and its limits. Continuous time Markov chains, Poisson process, pure birth process, birth and death processes, Markov Process with discrete states. Branching process, offspring distribution, extinction probabilities.

UNIT 4: Point estimation, general properties of estimators, Unbiasedness; strong, weak and squared error consistency, invariance property of consistent estimator, Fisher's measure of information, Cramer-Rao inequality. Efficiency, Likelihood function, Sufficiency, Exponential family of distributions, Rao-Blackwell and Lehmann-Scheffe theorems and their applications. Various methods of estimation-MLE, Scoring method, method of moments, minimum chi-square, modified minimum χ^2 , least-squares and properties of these estimators. Empirical distribution as an estimator of distribution function, location and scale family of distributions, Bayes estimator Prior and Posterior distributions, Neyman-Pearson Theory of Testing Hypotheses, Unbiasedness, UMPU, LMP, LMPU, tests of hypotheses concerning a real parameter, Likelihood ratio tests, asymptotic properties, Sequential procedures, SPRT-Wald's identity-OC and ASN functions, Confidence sets, shortest confidence intervals, interval estimation.

UNIT 5: Estimation of population mean and population variance under various methods of probability sampling designs with or without replacement, simple random sampling, PPS sampling, Stratified random sampling, systematic sampling, IPPS sampling, Midzuno-Sen scheme of sampling, Cluster sampling and two stage sampling. Various methods of allocation in stratified random sampling, post-stratification, applications of double sampling for stratified random sampling, Estimation using auxiliary information in survey sampling-ratio estimator, regression estimator, double sampling for ratio and regression method of estimation. General linear models, estimability of linear parametric functions, Gauss-Markov theorem, ANOVA-one-way classification, two-way classification with equal and unequal number of observations per cell, Standard designs: CRD, RBD, LSD, GLSD. Efficiency of design and comparison. Statistical analysis of symmetrical factorial designs. Total and partial confounding in 2^n, 3^n and p^n experiments. Incomplete block designs, BIBD, PBIBD. Split-plot and split-split plot designs, strip-plot design, missing and mixed plot analysis in RBD, LSD, GLSD.