University of Kerala

IV Semester B.Tech Degree Examination-April/May 2015

(2013 scheme)

13.401 Probability, Random Process and Numerical Techniques (FR)

Model Question Paper

Time: 3hours **Max.Marks:100**

Part A

Answer all questions. Each question carries 4 marks.

1. Find the constant c, so that $f(x) = c\left(\frac{2}{3}\right)^x$; $x = 1, 2, 3\ldots$ satisfies the p.d.f of a discreate random variable X.

2. If X is uniformally distributed random variable with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$.

3. Find the mean and variance of the stationary process with autocorrelation $R_X(\tau) = 16 + \dfrac{9}{1 + 6\tau^2}$

4. If $X(t)$ is a process in which $C(\tau) = qe^{-a|\tau|}$, show that $X(t)$ is mean-ergodic.

5. Construct Newton's forward interpolation polynomial for the following data:

x	:	1	2	3	4	5
y	:	5	11	19	29	41

Part B

Answer one full question from each module. Each question carries 20 marks.

Module I

6. (a) The distribution function of a random variable X is given by $F(X) = 1 - (1 + x)e^{-x}, x \geq 0$. Find its (i) p.d.f (ii) mean and (iii) variance.

   (b) Fit a Binomial distribution of the following data.

x	:	0	1	2	3	4	5
y	:	3	6	24	26	4	1

(c) The probability of hitting an aircraft is 0.001 for each shot. How many shots should be fired so that the probability of hitting with two or more shot is above 0.95? (use Poisson distribution)

7. (a) Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show a five or six.

(b) If X is normal variate with mean 30 and s.d 5. Find the probabilities that (i) $26 \leq X \leq 40$ (ii) $X \geq 45$ (iii) $|X - 30| > 5$

(c) The daily consumption of milk in excess of 20,000 gallons is approximately exponentially distributed with $\lambda = 3000$. The city has a daily stock of 35,000 gallons. What is the probability of 2 days selected at random, the stock's insufficient for both days ?

Module II

8. (a) The two dimensional random variable (X, Y) has the joint density function, $f(x, y) = \frac{1}{48}(x + 2y)$ where $x = 0, 1, 2$ and $y = 0, 1, 2, 3$. Find the conditional distribution of X and Y.

(b) The joint p.d.f of X and Y is given by $f(x, y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of k and prove also that X and Y are independent.

(c) Calculate the coefficient of correlation from the following data:

x	:	9	8	7	6	5	4	3	2	1
y	:	15	16	14	13	11	12	10	8	9

9. (a) Show that the process $X(t)$ such that

$$P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}} & n = 1, 2, \ldots \\ \frac{at}{1+at} & , n = 0 \end{cases} \text{ is not stationary.}$$

(b) If $X(t) = A\cos wt + B\sin wt$ and $Y(t) = B\cos wt - A\sin wt$ where A and B are independent random variables with $E(A) = E(B) = 0$ and $Var(A) = Var(B) = \sigma^2$, then show that $X(t)$ and $Y(t)$ are joint wide sense stationary.

Module III

10. (a) Define power spectrum density function. Find the power spectrum density of a random process if its autocorrelation function $R(t) = \begin{cases} 1 - \frac{|\tau|}{\tau}, & \text{if } |\tau| < T \\ 0 & \text{elsewhere} \end{cases}$

    (b) Show that the poisson process $\{X(t)\}$ given by the probability law $P\{X(t) = n\} = \dfrac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0, 1, 2 \ldots$ is not stationary.

11. (a) Find the power spectral density of a WSS process with auto correlationfunction $R(\tau) = e^{-\alpha\tau^2}$

    (b) Find the average power of the random process if $s(\omega) = \dfrac{10\omega^2 + 35}{(\omega^2 + 4)(\omega^2 + 9)}$

Module IV

12. (a) Solve $x^3 - 9x + 1 = 0$ for the root lying between 2 and 4 by regula falsi method correct to two decimals.

    (b) Apply Newton's interpolation formula to find $f(22)$ from the following data

x	:	20	24	28	32
$f(x)$	:	0.01427	0.01581	0.01772	0.01996

    (c) Find the approximate value of $\displaystyle\int_0^1 \frac{1}{1+x^2}dx$ by

    i. Trapezoidal rule and

    ii. Simpson's $\dfrac{1}{3}$ rule. Take $h = 0.1$.

13. (a) Find a root of $x^3 - x - 11 = 0$ that lies between 2 and 3 correct to two decimal places by bisection method.

    (b) By using Newton-Raphson's method find the root of $x^4 - x - 10 = 0$ which near to $x = 2$ correct to three places of ecimal.

    (c) Find the value of y when $x = 10$, given that

x	:	5	6	9	11
y	:	12	13	14	16