UNIVERSITY OF KERALA

FIRST DEGREE PROGRAMME IN MATHEMATICS

Model Question Paper

Semester 1 MM 1141 Methods of Mathematics

Time: Three hours | Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. What is Fermat number?
2. Convert 1536_{10} to base 2.
3. What is the gcd of $2^3 5^4 7$ and $2^2 5^6 7^3$.
4. Find the smallest non negative residue of $31 \pmod{25}$.
5. Find the natural domain of the function $f(x) = \dfrac{x}{|x|}$.
6. Find $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$.
7. Find $\dfrac{dy}{dx}$ if $y = x^x$.
8. Is the origin a point of inflexion for the curve $f(x) = x^4$?
9. What kind of asymptotes will an equilateral hyperbola have?
10. Find the centre of the hyperbola $x^2 - y^2 - 4x + 8y - 21 = 0$.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Consider the relation $\approx$ on $\mathbb{Z}$ defined by $a \approx b$ if and only if $a \geq b$. Will it be an equivalance relation? If so, what are its equivalence classes? If not, determine which properties of an equivalence relation fail.
12. Prove that there are infinitely many primes.
13. Find all solutions of the equation $365x + 1876y = 24$.
14. Show that $2^{560} \equiv 1 (\bmod 561)$.
15. Given numbers a and b, suppose there are integers r and s so that $ar + bs = 1$.
    Show that a and b are coprime.
16. Check whether the congruence $12x \equiv 5 (\bmod 25)$ has a solution.
    If so, find the least non-negative solution.

17. Consider the function $f(x) = \sin x - \cos x$ on $[0, 2\pi]$.

    (a) Express $f'(x)$ as a sine function.

    (b) Determine the interval in which f is decreasing.

18. Find the value of the constant k, if possible, that will make the function

    $$f(x) = \begin{cases} kx^2 & \text{if} \quad x \le 2 \\ 2x + k & \text{if} \quad x > 2 \end{cases}$$

    continuous everywhere.

19. Given that the tangent line to $y = f(x)$ at the point $(-1, 3)$ passes through the point $(0, 4)$, find $f'(-1)$.

20. Given $f'(x) = \sqrt{3x+4}$ and $g(x) = x^2 - 1$, find $F'(x)$ if $F(x) = f(g(x))$.

21. Let $f(x) = \tan x$.

    (a) Show that there is no number c such that $f'(c) = 0$, even though $f(0) = f(\pi) = 0$.

    (b) Explain why the result in part (a) does not violate Rolle's Theorem.

22. Define an ellipse. State the reflection property of ellipses

Section III

Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. Define equivalence relation on a set S. Show that congruence modulo m is an equivalence relation in the set of integers.

24. Find the greatest common divisor d of $a = 267$ and $b = 112$. Find also integers r and s so that $ar + bs = d$.

25. Given integers a, b, c, show that there exist integers m and n with $am + bn = c$ if and only if (a, b) divides c. Use this result to solve the equation $98 = 1001x + 840y$.

26. Sketch the graph of $y = 2 - \dfrac{1}{x+1}$ by transforming the graph of $y = \dfrac{1}{x}$ appropriately.

27. Find the minimum distance of the point $(4, 2)$ from the parabola $y^2 = 8x$.

28. A particle moves along a straight line so that after t minutes its distance is $s = 6t^4$ ft from the origin.

    (a) Find the average velocity of the particle over the interval $[2, 4]$.

    (b) Find the instantaneous velocity at t=2 minutes.

29. Consider the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

    (a) Find $\dfrac{dy}{dx}$.

    (b) Write down the equation of the tangent to the curve at (x_0, y_0).

(c) Express the equation of the tangent in the intercept form and thereby find the length of the tangent intercepted between the axes.

30. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

31. Find the foci, centre and the equations of the directrices of the ellipse $2x^2 + 3y^2 = 1$.

Section IV

Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. (a) Prove that $ax \equiv 1 \pmod{m}$ has a solution if and only if $(a, m) = 1$.

(b) Using induction, prove that for all $n \geq 1$, 5 divides $3^{4n} - 1$.

(c) State and prove the Division Theorem.

33. (a) Show that the function $f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 1 \\ x + 2 & \text{if } x > 1 \end{cases}$ is continuous, but not differentiable at $x = 1$.

(b) A particle is moving along a curve whose equation is $\dfrac{xy^3}{1+y^2} = \dfrac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units /sec when the particle is at the point (1,2). At what rate is the y- coordinate of the point changing at that instant?

(c) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

34. (a) Use Squeezing Theorem to prove that $\lim\limits_{x \to 0} x \sin\left(\dfrac{1}{x}\right) = 0$.

(b) Find the absolute maximum and absolute minimum of the function $f(x) = 1 + |9 - x^2|$.

(c) Use the intermediate theorem to show that there is a square with diagonal length between r and $2r$ and area half that of a circle of radius r.

35. (a) Suppose the axes of an xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates.

(b) Identify and sketch the conic $153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$.

UNIVERSITY OF KERALA

FIRST DEGREE PROGRAMME IN PHYSICS

Model Question Paper

Semester 1 MM 1131.1 Complementary Course for Physics

Mathematics I: Differentiation and Analytic Geometry

Time: Three hours | Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. If a particle moves with a constant velocity, what can you say about its position versus time curve?
2. Define the jump of a function.
3. Explain the convergence of the harmonic series.
4. Write the Maclaurin series expansion of e^x.
5. Find the degree of homogeneity of the function $f(x,y) = \sin^{-1}\left(\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)$.
6. Write down the condition that the line $y = mx + c$ be a tangent to the parabola $y^2 = 8x$.
7. State the rotation equations in Cartesian coordinates.
8. State Kepler's third law.
9. Find the slopes of the asymptotes of the rectangular hyperbola $y^2 - x^2 = 1$.
10. Write down the polar equation of a circle.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Find the natural domain of the function $\sqrt{x^2 - 5x + 6}$.
12. Find k so that $f(-3) = \lim\limits_{x\to -3} f(x)$ where $f(x) = \begin{cases} \dfrac{x^2-9}{x+3} & \text{if} \quad x \neq -3 \\ k \quad \text{if} \quad x = -3 \end{cases}$
13. Find $\dfrac{d^2y}{dx^2}$ where $x = \sec^2 t$ and $y == \tan^2 t$.
14. Find the slope of the curve $y = x^2 + 1$ at the point $(2, 5)$ and use it to find the equation of the tangent at $x = 2$.
15. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.
16. Test the convergence of the series $\dfrac{3}{10} + \dfrac{3}{10^2} + \dfrac{3}{10^3} + \cdots$ and find the sum if it is convergent.

17. Let $T = x^2y - xy^3 + 2, x = r\cos\theta, y = r\sin\theta$. Find $\dfrac{\partial T}{\partial r}$ and $\dfrac{\partial T}{\partial \theta}$.

18. Determine whether the series $\sum\limits_{k=1}^{\infty}\left(\dfrac{1}{k} - \dfrac{1}{k+2}\right)$ converges or diverges. If it converges find the sum?

19. Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ if $x = \dfrac{2u}{u^2+v^2}$ and $y = \dfrac{-2v}{u^2+v^2}$.

20. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

21. Verify Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x, \quad x \in [0,2]$.

22. Find the equation of the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$.

Section III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. (a) Suppose that $z = x^3y^2$, where both x and y are changing with time. At a certain instant when $x = 1$ and $y = 2$, x is decreasing at the rate of 2 units/s, and y is increasing at the rate of 3 units/s. How fast is z changing at this instant? Is z increasing or decreasing?

    (b) Find the slope of the tangent at $(-1, 1, 5)$ to the curve of intersection of the surface $z = x^2 + 4y^2$ and the plane $x = -1$.

24. Locate all relative extrema and saddle points of the function $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.

25. A particle is moving along the curve whose equation is $\dfrac{xy^3}{1+y^2} = \dfrac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at $(1, 2)$

    (a) At what rate is the y-coordinate of the point changing at that instant?

    (b) Is the particle rising or falling at that instant?

26. A glass of lemonade with a temperature of 40°F is left to sit in a room whose temperature is a constant 70°F. Using Newton's law of cooling, show that if the temperature of the lemonade reaches 52°F in 1 hour, then the temperature T of the lemonade as a function of the elapsed time t is modelled by the equation

$$T = 70 - 30e^{-0.5t}$$

where T is in °F and t is in hours. Draw the graph taking t in horizontal direction and T in vertical direction.

    (a) What happens to the rate of temperature rise over time.

    (b) Use a derivative to confirm your conclusion in part (a).

    (c) Find the average temperature of the lemonade over the first 5 hours.

27. Find the Taylor series expansion for $\tan^{-1} x$ about $x = 1$.

28. Find the absolute maximum and absolute minimum of $f(x,y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices $(0,0), (3,0), (0,5)$.

29. Identify and sketch the curve $xy = 1$.

30. Identify and sketch the curve $153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$

31. The period T of a simple pendulum with small oscillations is calculated from the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum and g is the acceleration due to gravity. Suppoe that measured values of L and g have errors of at most 0.5% and 0.1% respectively. Use differentials to approximate the maximum percentage error in the calculated value of T.

Section IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (a) State L'-Hospital's rule. Evaluate

    i. $\lim\limits_{x \to \frac{\pi}{2}} (\cos x)^{\tan x}$.

    ii. Evaluate $\lim\limits_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$.

    (b) Given an electrical circuit consisting of an electromotive force that produces a voltage V, a resistor with resistance R, and an inductor with inductance L.

    It is shown in electrical circuit theory that if the voltage is first applied at time $t = 0$ then the current I flowing through the circuit at time t is given by $I = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$. What is the effect of the current at a fixed time t if the resistance approaches 0?

    (c) A closed cylindrical can is to hold 1 litre of liquid. How should we choose the height and radius to minimize the amount of material needed to manufacture the can?

33. (a) Determine the dimensions of a rectangular box open at the top, having a volume 32 cubic feet and requiring the least material for its construction.

    (b) Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.

34. (a) Prove that the line tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) has the equation $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$.

    (b) A line tangent to the hyperbola $4x^2 - y^2 = 36$ intersects the y-axis at the point $(4, 0)$. Find the point(s) of tangency.

    (c) The orbit of Halley's comet (last seen in 1986) has an eccentricity of 0.97 and a semimajor axis given by $a = 18.1$ AU.

    (i) Find the equation of its orbit in the polar coordinate system.

    (ii) Find the period of its orbit.

    (iii) Find its perihelion and aphelion distances.

35. (a) Find the equation of the asymptotes of the curve $x^2 - y^2 - 4x + 8y - 21 = 0$

    (b) Sketch the graph of $r = \frac{2}{1 + 2\sin\theta}$ in polar coordinates.

UNIVERSITY OF KERALA

First Degree Programme in Chemistry

Model Question Paper

Semester 1 MM 1131.2 Complementary Course for Chemistry

Mathematics I: Differentiation and Matrices

Time: Three hours — Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. What is the geometric interpretation of the average velocity?
2. On which of the folllowing intervals is the function $f(x) = \dfrac{1}{\sqrt{x-2}}$ continuous?
   (a) $[2, +\infty)$, (b) $(-\infty, +\infty)$, (c) $(2, +\infty)$, (d) $[1, 2)$.
3. Find $f'(x)$ if $f(x) = \dfrac{\csc x}{\cot x}$.
4. State the extreme value theorem.
5. What is the sum of the series $\displaystyle\sum_{k=0}^{\infty} \frac{5}{4^k}$?
6. Find the degree of homogeneity of the function $f(x, y) = \sqrt{x^2 + y^2}$.
7. When do we say that a system of equations is consistent?
8. Find the rank of the matrix $\begin{pmatrix} -3 & 1 \\ 2 & 2 \\ 4 & -3 \end{pmatrix}$
9. Define similar matrices.
10. Check whether the matrix $\begin{pmatrix} 1 & 1 & -3 \\ 2 & 16 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ is singular.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Find the parametric equations of the circle of radius 5 units, centred at the origin, oriented clockwise.
12. Given $f(x) = x^2 + 1$ and $g(x) = 3x + 2$, find all values of x such that $f(g(x)) = g(f(x))$.
13. Use implicit differentiation to show that the equation of the tangent line to the curve $y^2 = kx$ at (x_0, y_0) is $y_0 y = \frac{1}{2}k(x + x_0)$.
14. A rock thrown downward with an unknown initial velocity from a height of 1000 ft reaches the ground in 5 seconds. Use the formula $s = s_0 + vt + \frac{1}{2}gt^2$ to find the velocity of the rock when it hits the ground.

15. Evaluate the limits

    (a) Find $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$

    (b) Find $\lim\limits_{x \to \pi} (x-\pi)\cot x$

16. Find f_x if $f(x,y) = \dfrac{x+y}{x-y}$.

17. What is the velocity interpretation of the Mean Value Theorem?

18. Use the formula for the binomial series to obtain the Maclaurin series for $f(x) = \dfrac{1}{(1+x)^3}$.

19. A point moves along the intersection of the elliptic paraboloid $z = x^2 + 3y^2$ and the plane $y = 1$. At what rate is z changing with x when the point is at $(2, 1, 7)$?

20. What are the various elementary row operations?

21. When do we say that a matrix is in row reduced echelon form?

22. What are eigen values and eigen vectors associated with a matrix?

Section III

Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. (a) State the Mean Value Theorem.

    (b) Let $f(x) = x^{\frac{2}{3}}$, $a = -1$ and $b = 8$.

    (i) Show that there is no number c in (a, b) such that

    $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

    (ii) Explain why the result in part (a) does not violate the Mean Value Theorem.

24. (a) Use a chain rule to find the value of $\dfrac{dz}{dt}$ at $t = 3$ if $z = x^2 y$; $x = t^2$; $y = t + 7$.

    (b) Let f be a differentiable function of one variable, and let $z = f(x^2 + y^2)$. Show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$.

25. Let A be the area of a circle of radius r, and assume that r increases with the time t.

    (a) Draw a picture of the circles with the labels A and r placed appropriately.

    (b) Write an equation that relates A and r.

    (c) Use the equation in part (b) to to find an equation that relates $\dfrac{dA}{dt}$ and $\dfrac{dr}{dt}$.

    (d) At a certain instant the radius is 5 cm and increasing at the rate of 2 cm/s. How fast is the area increasing at that instant?

26. The formula $F = \frac{9}{5}C + 32$, where $C \geq -273.15$ expresses the Fahrenheit temperature F as a function of the Celsius temperature C.

    (a) Find a formula for the inverse function.

(b) In words, what does the inverse function tell you?

(c) Find the domain and range of the inverse function.

27. The temperature at a point (x, y) on a metal plate in the xy plane is $T(x, y) = x^3 + 2y^2 + x$ degrees Celsius. Assume that distance is measured in centimetres and find the rate at which temperature changes with respect to distance if we start at the point $(1, 2)$ and move

(a) to the right and parallel to the x-axis.

(b) upward and parallel to the y-axis.

28. Use the method of Lagrangian multipliers to find the points on the circle $x^2 + y^2 = 45$ that are closest to and farthest from the point $(1, 2)$.

29. Solve the system of equations

$$\begin{aligned} x_1 - 7x_2 + 6x_4 &= 5 \\ x_3 - 2x_4 &= -3 \\ -x_1 + 7x_2 - 4x_3 + 2x_4 &= 7 \end{aligned}$$

30. Find the row reduced echelon form of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \\ 4 & 0 & 7 \end{pmatrix}$ and determine its rank.

31. Find the eigen values of the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and the eigen vector(s) corresponding to the largest eigen value.

Section IV

Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. (a) A wire of length 12 in can be bent into a circle, bent into a square, or cut into two pieces to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be a maximum?

(b) Using Lagrange multipiers find the maximum and minimum values of $f(x, y) = xy$ subject to $4x^2 + 8y^2 = 16$.

33. (a) Express the number 10 as the sum of two non-negative numbers whose product is as large as possible.

(b) According to the ideal gas law, the pressure, temperature and volume of a gas are related by $P = kT/V$, where k is a constant of proportionality. Suppose that V is measured in cubic inches (in^3, T is measured in kelvins (K), and that for a certain gas the constant of proportionality is $k = 10$ in.lb/K.

(i) Find the instanteneous rate of change of pressure with respect to temperature if the temperature is 80 K and the volume remains fixed at 50 in^3.

(ii) Find the instanteneous rate of change of volume with respect to pressure if the volume is 50 in^3 and 80 K and the temperature remains fixed at 80 K.

34. (a) Use a total differential to approximate the change in $f(x, y) = \dfrac{xyz}{x + y + z}$ as (x, y) varies from $P(-1, -2, 4)$ to $Q(-1.04, -1.98, 3.97)$. Compare your estimate with the actual change in $f(x, y)$.

(b) Diagonalise the matrix $\begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{pmatrix}$

35. (a) Find the general solution of the system

$$\begin{aligned} 2x_1 - 3x_2 &= 1 \\ -x_1 + 3x_2 &= 0 \\ x_1 - 4x_2 &= 3 \end{aligned}$$

or show that the system has no solution.

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ -4 & -4 & 5 \end{pmatrix}$

UNIVERSITY OF KERALA

First Degree Programme in Geology

Model Question Paper

Semester 1 MM 1131.3 Complementary Course for Geology

Mathematics I: Differentiation and Theory of Equations

Time: Three hours | Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. If a particle moves with a constant velocity, what can you say about its position versus time curve?
2. Define the jump of a function.
3. Explain the convergence of the harmonic series.
4. Write down the Maclaurin series for $\ln(1+x)$.
5. If $z = e^{2x}\sin y$, find $\dfrac{\partial z}{\partial y}$.
6. State the degree of homogeneity of the function $f(x, y) = x^2y - y^3$.
7. State the Fundamental Theorem of Algebra.
8. State the Descarte's rule of signs.
9. Define a reciprocal equation.
10. Construct an equation whose roots are 1, 2 and 3.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Find the natural domain of the function $\sqrt{x^2 - 5x + 6}$.
12. Find k so that $f(-3) = \lim\limits_{x\to -3} f(x)$ where $f(x) = \begin{cases} \dfrac{x^2-9}{x+3} & \text{if} \quad x \neq -3 \\ k \quad \text{if} \quad x = -3 \end{cases}$
13. Find $\dfrac{d^2y}{dx^2}$ where $x = \sec^2 t$ and $y == \tan^2 t$.
14. Find the slope of the curve $y = x^2 + 1$ at the point $(2, 5)$ and use it to find the equation of the tangent at $x = 2$.
15. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.
16. Test the convergence of the series $\dfrac{3}{10} + \dfrac{3}{10^2} + \dfrac{3}{10^3} + \cdots$ and find the sum if it is convergent.

17. Let $T = x^2y - xy^3 + 2, x = r\cos\theta, y = r\sin\theta$. Find $\dfrac{\partial T}{\partial r}$ and $\dfrac{\partial T}{\partial \theta}$.

18. Determine whether the series $\sum\limits_{k=1}^{\infty} \left(\dfrac{1}{k} - \dfrac{1}{k+2}\right)$ converges or diverges. If it converges find the sum?

19. Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ if $x = \dfrac{2u}{u^2+v^2}$ and $y = \dfrac{-2v}{u^2+v^2}$.

20. Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$ if two of the roots of the equation are equal inmagnitude and opposite in sign.

21. Find the number and position of the real roots of the equation $x^3 + x^2 - 2x - 1 = 0$.

22. Form the equation whose roots are the roots of the equation $x^3 - 6x^2 + 12x - 8 = 0$, diminished by 2.

Section III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. (a) Suppose that $z = x^3y^2$, where both x and y are changing with time. At a certain instant when $x = 1$ and $y = 2$, x is decreasing at the rate of 2 units/s, and y is increasing at the rate of 3 units/s. How fast is z changing at this instant? Is z increasing or decreasing?

    (b) Find the slope of the tangent at $(-1, 1, 5)$ to the curve of intersection of the surface $z = x^2 + 4y^2$ and the plane $x = -1$.

24. Locate all relative extrema and saddle points of the function $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.

25. A particle is moving along the curve whose equation is $\dfrac{xy^3}{1+y^2} = \dfrac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at $(1, 2)$

    (a) At what rate is the y-coordinate of the point changing at that instant?

    (b) Is the particle rising or falling at that instant?

26. Find the Taylor series expansion for $\tan^{-1} x$ about $x = 1$.

27. Find the absolute maximum and absolute minimum of $f(x,y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices $(0,0), (3,0), (0,5)$.

28. Let $f(x) = x^2 + px + q$. Find the values of p and q such that $f(1) = 3$ is an extreme value of f on $[0, 2]$. Is this value a maximum or a minimum?

29. Use the method of Lagrangian multipliers to find the maximum and minimum values of the function $f(x,y) = xy$ subject to the constraint $4x^2 + 8y^2 = 16$.

30. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find the values of $\Sigma\alpha^2$, $\Sigma\alpha^3$, $\Sigma\alpha^2\beta$ and $\Sigma\alpha^2\beta^2$.

31. Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

Section IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (a) A projectile is launched upward from ground level with an initial speed of 60 m/s.

    (i) How long does it take for the projectile to reach its highest point?

    (ii) How high does the projectile go?

    (iii) How long does it take for the projectile to drop back to the ground from its highest point?

    (iv) What is the speed of the projectile when it hits the ground?

    (b) A closed cylindrical can is to hold 1 litre of liquid. How should we choose the height and radius to minimize the amount of material needed to manufacture the can?

33. (a) Determine the dimensions of a rectangular box open at the top, having a volume 32 cubic feet and requiring the least material for its construction.

    (b) Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.

34. (a) Solve, using Cardano's method, the equation $x^3 - 15x - 126 = 0$.

    (b) Solve, using Ferrari's method, the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

35. (a) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace's equation.

    (b) Given $z = 8x^2y - 2x + 3y$, $x = uv$ and $y = u - v$, use the chain rule to find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

    (c) Let f be a differentiable function of one variable and let $z = f(x^2 + y^2)$. Show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$.

UNIVERSITY OF KERALA

FIRST DEGREE PROGRAMME IN STATISTICS

Model Question Paper

Semester 1 MM 1131.4 Complementary course for Statistics

Mathematics I: Theory of Equations, Analytic Geometry and Infinite series

Time: Three hours | Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. State the Fundamental Theorem of Algebra.
2. State Descarte's rule of signs.
3. Define a reciprocal equation.
4. Define a monotone sequence.
5. For what value of r is the geometric series $ar + ar^2 + ar^3 + \cdots$ convergent?
6. What is the value of $\lim_{n\to\infty} \frac{n}{n+1}$?
7. What is the eccentricity of a parabola.
8. What are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?
9. What is the parametric equation of the rectangular hyperbola?
10. Write down the condition for the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a pair of lines.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Solve the equation $x^4 + x^2 - 2x + 6 = 0$, given $1 + i$ is a root.
12. Solve the equation $2x^3 - x^2 - 18x + 9 = 0$, if two of the roots are equal in magnitude but opposite in sign.
13. Find the number of positive real roots of the equation $x^4 - 6x^3 + 10^2 - 6x + 1 = 0$.
14. Diminish by 2 the roots of the equation $2x^3 - 7x^2 + 3x - 5 = 0$.
15. Define a divergent sequence and give an example.
16. State Leibnitz's test for the covergence of an alternating series.
17. Show that $\lim_{n\to\infty} \left(\frac{1+2+3+\cdots+n}{n^2}\right) = \frac{1}{2}$.
18. Define the orthoptic locus of a conic.

19. Define a rectangular hyperbola and deduce its equation.

20. Find the equation of tangent at θ on the parabola $y^2 = 4ax$.

21. Determine the nature of the roots of the equation $x^4 + 3x^2 + 2x - 7 = 0$.

22. Test for convergence the series

$$\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \ldots + \frac{n^2}{2^n} + \ldots$$

Section III

Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\alpha\beta + \beta\gamma$, $\beta\gamma + \gamma\alpha$ and $\gamma\alpha + \alpha\beta$.

24. Solve the reciprocal equation $6x^4 + 35x^3 + 62x^2 + 35x + 6 = 0$.

25. Test for the convergence of the series : $\dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \cdots$

26. Show that the sequence $\left\{\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \cdots \dfrac{n}{n+1}, \cdots\right\}$ is strictly monotone and bounded.

27. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

28. Find the equation of the chord joining the points θ and ϕ on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

29. Find the equation of the normal at the point t on the rectangular hyperbola $xy = c^2$.

30. Prove that tangents at the ends of focal chords of a parabola intersect at right angles on the directrix.

31. Find the coordinates of all points on the hyperbola $4x^2 - y^2 = 4$ where the two lines that pass through the point and the foci are perpendicular.

Section IV

Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. (a) Solve by Ferrari's method: $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

    (b) Find the number and position of the real roots of the equation $x^3 - 3x + 1 = 0$.

    (c) Solve by Cardano's method the equation: $x^3 - 18x - 35 = 0$.

33. (a) Test the convergence of the series : $\sum\limits_{k=1}^{\infty} \dfrac{(2k)!}{4^k}$ using ratio test.

    (b) Test the convergence of the alternating series $\sum\limits_{k=1}^{\infty} \dfrac{(-1)^{k+1}(k+3)}{k(k+1)}$.

    (c) Show that the sequence (a_n) where $a_n = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots + \dfrac{1}{n!}$ is convergent and find its limit.

34. (a) Suppose the axes of an xy-coordinate system are rotated through an angle of $\theta = 45°$ to obtain an $x'y'$-coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$-coordinates.

    (b) Identify and sketch the conic $153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$.

35. Let an $x'y'$- coordinate system be obtained by rotating the xy-coordinate system through an angle $\theta = 60^0$.

    (a) Find the $x'y'$- coordinates of the point whose xy-coordinates are $(-2, 6)$.

    (b) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$- coordinates.

    (c) Sketch the curve in part (b), showing both xy-axes and $x'y'$-axes.

UNIVERSITY OF KERALA

FIRST DEGREE PROGRAMME IN ECONOMICS

Model Question Paper

Semester 1 MM 1131.5 Complementary course for Economics

Mathematics for Economics I

Time: Three hours | Maximum marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. For the demand function $x = \dfrac{90}{p+5} - 6$, at what price does the demand tend to vanish?
2. Find $\lim\limits_{x \to \frac{1}{2}} \dfrac{x^n - \left(\frac{1}{2}\right)^n}{x - \frac{1}{2}}$.
3. Find the derivative of $y = x^5(2x^2 + 1)$.
4. If the total cost function is $\pi = ax^2 + bx + c$ find the marginal cost.
5. When do we say that a function f is differentiable at a point?
6. Give an example of a function that is contiuous but not differentiable at a point.
7. Obtain a relation between the variables x and y if it is given that $x = t^2$ and $y = 2t$.
8. What is the geometrical interpretation of the derivative of a function at a point?
9. Write down the derivative of $\log_a x$.
10. What is the value of $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$?

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Find the natural domain of the function $\sqrt{x^2 - 5x + 6}$.
12. Draw the graph of the function $xy = 3$.
13. Examine the demand curve $p = \dfrac{a}{ax+b}$ where a and b are positive constants. Show that the demand increases from zero to indefinitely large amounts as the price falls. What type of curve is the total revenue curve?
14. For the curve $q = 30 - 4p - p^2$, find the elasticity of demand η for $p = 3$.
15. Find $\dfrac{dy}{dx}$ if
    (a) $xy + y^2 = 4$
    (b) $x = t + 1, y = t^2 + 1$

16. Differentiate with respect to x

   (a) $y = \dfrac{x^3}{x^2+1}$

   (b) $y = \dfrac{1}{\sqrt{x+1}}$

17. Discuss the continuity of the function $f(x) = \sqrt{9-x^2}$.

18. For what values of x there is a gap in the graph of $y = \dfrac{x^2-9}{x^2-5x+6}$.

19. If the demand law is $p = \dfrac{a}{x} - c$, show that the total revenue decreases as output increases, marginal revenue being a negatve constant.

20. Find the slope of the tangent to the curve $y = ax + b + \dfrac{c}{x}$ at the point with abscissa x_1. Where is the tangent parallel to OX?

21. Find $\dfrac{dy}{dx}$ if

   (a) $y = (x^2+1)\ln(x^2+1)$

   (b) $y = \ln x^2$

22. If x and y satisfy the relation $x^2 + y^2 = 1$, show that $\dfrac{dy}{dx} = -\dfrac{x}{y}$.

Section III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. What are the requirements for the continuity of a function? Find k so that

$$f(x) = \begin{cases} \dfrac{x^2-9}{x+3} & \text{if} \quad x \neq -3 \\ k & \text{if} \quad x = -3 \end{cases}$$

is a continuous function.

24. Plot a graph of $y = \dfrac{2x}{x^2+1}$ for positive values of x, and show how the graph for negative values of x can be deduced. What are the greatest and least values of y?

25. If $y = x^4 - 4x^3 + 6x^2 - 4x - 3$, show that $\dfrac{dy}{dx} = 4(x-1)^3$.

26. If $f(x)$ is a single-valued function of x, express the derivatives of $\sqrt{f(x)}$ and its reciprocal interms of the derivative of $f(x)$.

27. Find the function inverse to $y = \dfrac{2x+1}{x-1}$ and show that it is single-valued. Represent it graphically and give some account of the behaviour of the graph in the neighbourhood of $x = 1$ and of $y = 2$.

28. Show that marginal rvenue can always be expressed as $p + x\dfrac{dp}{dx}$. Deduce that the gradient of the demand curve is numerically equal to $\dfrac{p}{x}$ at the output where marginal revenue is zero.

29. Examine the function $y = \dfrac{1-x}{1+x}$ from the point of view of continuity and illustrate it graphically.

30. Discuss briefly the various cases of the limits of $f(x)$ as $x \to \pm\infty$.

31. If $f(x) = 4 + \dfrac{1}{1 + \dfrac{1}{(1-x)}}$, $0 < x < 1$, find $\lim\limits_{x \to 1} f(x)$.

Section IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (a) If $f(x) = x^2$, $g(x) = \sin x$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

(b) Find the domain and range of the function $y = \dfrac{1+x}{1-x}$.

(c) If $f(x) = \dfrac{x^2 + 3x - 2}{x^2 + 2x + 4}$, express $f(2a)$ in terms of a.

33. (a) Given the function $f(x) = \begin{cases} 5 - x & \text{if} \quad x \neq 4 \\ 0 & \text{if} \quad x = 4 \end{cases}$

(i) Draw a graph of the function.

(ii) Identify the discontinuity of the function in the graph.

(iii) Find $\lim\limits_{x \to 4} (5 - x)$ and show that the value of the limit is not equal to the value of the function at $x = 4$. What do you conclude?

(b) From the function $xy + 2x + y - 1 = 0$. Find the limit of y as $x \to -1$, and the limit of x as $y \to 1$. What restriction must be added to the statement that y is a continuous function of x, and conversely.

34. (a) Explain the concepts of the total revenue curve, average and marginal revenue curves.

(b) The number (x) of persons per day using a motor coach service to Southend is related to the fare (p shillings) charged by the law $p = \left(3 - \dfrac{x}{40}\right)^2$. Show that the demand curve is a parabola and locate its vertex. Also graph the total revenue curve, showing that revenue rises rapidly to a maximum before falling off slowly.

35. Find $\dfrac{dy}{dx}$ if

(a) $y = \dfrac{e^x \ln x}{x^2}$

(b) $y = \sqrt{\dfrac{1+x}{1-2x}}$

(c) $x^2 + y^2 + 2x + 4y + 5 = 0$

UNIVERSITY OF KERALA

First Degree Programme in Physics and Computer Applications

Model Question Paper

Semester 1 MM 1131.6 Complementary Course in Mathematics
(Complex Numbers, Differentiation and Theory of Equations)

Time: Three hours Maximum Marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. If ω is a complex n^{th} root of unity, what is the value of $\omega^{2013} + (\omega^2)^{2013}$?
2. State de Moivre's theorem.
3. If a particle moves with a constant velocity, what can you say about its position versus time curve?
4. Explain the convergence of the harmonic series.
5. Write down the Maclaurin series for $\ln(1 + x)$.
6. If $z = e^{2x} \sin y$, find $\dfrac{\partial z}{\partial y}$.
7. State the degree of homogeneity of the function $f(x, y) = x^2 y - y^3$.
8. State the Fundamental Theorem of Algebra.
9. Define a reciprocal equation.
10. Construct an equation whose roots are 1, 2 and 3.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Prove that $\left(\dfrac{1 + \cos\theta + i \sin\theta}{1 + \sin\theta + i \cos\theta}\right)^n = \cos n\theta + i \sin n\theta$, where n is a positive integer.
12. Find all the values of $(1 + i)^{\frac{1}{4}}$.
13. Find the natural domain of the function $\sqrt{x^2 - 5x + 6}$.
14. Find the slope of the curve $y = x^2 + 1$ at the point $(2, 5)$ and use it to find the equation of the tangent at $x = 2$.
15. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.
16. Test the convergence of the series $\dfrac{3}{10} + \dfrac{3}{10^2} + \dfrac{3}{10^3} + \cdots$ and find the sum if it is convergent.
17. Let $T = x^2 y - xy^3 + 2, x = r \cos\theta, y = r \sin\theta$. Find $\dfrac{\partial T}{\partial r}$ and $\dfrac{\partial T}{\partial \theta}$.

18. Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{1}{k}-\frac{1}{k+2}\right)$ converges or diverges. If it converges find the sum?

19. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ if $x=\frac{2u}{u^2+v^2}$ and $y=\frac{-2v}{u^2+v^2}$.

20. Solve the equation $x^3-5x^2-4x+20=0$ if two of the roots of the equation are equal inmagnitude and opposite in sign.

21. Find the number and position of the real roots of the equation $x^3+x^2-2x-1=0$.

22. Form the equation whose roots are the roots of the equation $x^3-6x^2+12x-8=0$, diminished by 2.

Section III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. Express $\sin^4\theta\cos^2\theta$ in terms of cosines of multiples of θ.

24. Separate $\tan^{-1}(\alpha+i\beta)$ into real and imaginary parts.

25. Locate all relative extrema and saddle points of the function $f(x,y)=3x^2-2xy+y^2-8y$.

26. A particle is moving along the curve whose equation is $\frac{xy^3}{1+y^2}=\frac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at $(1,2)$

    (a) At what rate is the y-coordinate of the point changing at that instant?

    (b) Is the particle rising or falling at that instant?

27. Find the Taylor series expansion for $\tan^{-1}x$ about $x=1$.

28. Find the absolute maximum and absolute minimum of $f(x,y)=3xy-6x-3y+7$ on the closed triangular region R with vertices $(0,0),(3,0),(0,5)$.

29. Use the method of Lagrangian multipliers to find the maximum and minimum values of the function $f(x,y)=xy$ subject to the constraint $4x^2+8y^2=16$.

30. If α, β, γ are the roots of the equation $x^3-px^2+qx-r=0$, find the values of $\Sigma\alpha^2$, $\Sigma\alpha^3$, $\Sigma\alpha^2\beta$ and $\Sigma\alpha^2\beta^2$.

31. Solve the equation $4x^4-20x^3+33x^2-20x+4=0$.

Section IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (a) Express $\cos 6\theta$ in terms of powers of $\cos\theta$.

    (b) If $\cos(x+iy)=\cos\theta+i\sin\theta$, show that $\cos 2x+\cosh 2y=2$.

33. (a) Determine the dimensions of a rectangular box open at the top, having a volume 32 cubic feet and requiring the least material for its construction.

    (b) Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.

34. (a) Solve, using Cardano's method, the equation $x^3 - 15x - 126 = 0$.

    (b) Solve, using Ferrari's method, the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.

35. (a) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace's equation.

    (b) Given $z = 8x^2y - 2x + 3y$, $x = uv$ and $y = u - v$, use the chain rule to find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

    (c) Let f be a differentiable function of one variable and let $z = f(x^2 + y^2)$. Show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$.

UNIVERSITY OF KERALA

FIRST DEGREE PROGRAMME IN CHEMISTRY AND INDUSTRIAL CHEMISTRY

Model Question Paper

Semester 1 MM 1131.7 Complementary Course in Mathematics
(Complex Numbers, Differentiation and Matrices)

Time: Three hours | Maximum Marks : 80

Section I

All the first ten questions are compulsory. They carry 1 mark each.

1. If ω is a complex n^{th} root of unity, what is the value of $\omega^{2013} + (\omega^2)^{2013}$?
2. State de Moivre's theorem.
3. If a particle moves with a constant velocity, what can you say about its position versus time curve?
4. Explain the convergence of the harmonic series.
5. Write down the Maclaurin series for $\ln(1 + x)$.
6. If $z = e^{2x}\sin y$, find $\dfrac{\partial z}{\partial y}$.
7. State the degree of homogeneity of the function $f(x, y) = x^2y - y^3$.
8. Find the rank of the matrix $\begin{pmatrix} -3 & 1 \\ 2 & 2 \\ 4 & -3 \end{pmatrix}$
9. Define similar matrices.
10. Check whether the matrix $\begin{pmatrix} 1 & 1 & -3 \\ 2 & 16 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ is singular.

Section II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Prove that $\left(\dfrac{1 + \cos\theta + i\sin\theta}{1 + \sin\theta + i\cos\theta}\right)^n = \cos n\theta + i\sin n\theta$, where n is a positive integer.
12. Find all the values of $(1 + i)^{\frac{1}{4}}$.
13. Find the natural domain of the function $\sqrt{x^2 - 5x + 6}$.
14. Find the slope of the curve $y = x^2 + 1$ at the point $(2, 5)$ and use it to find the equation of the tangent at $x = 2$.
15. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.

16. Test the convergence of the series $\frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \cdots$ and find the sum if it is convergent.

17. Let $T = x^2y - xy^3 + 2, x = r\cos\theta, y = r\sin\theta$. Find $\frac{\partial T}{\partial r}$ and $\frac{\partial T}{\partial \theta}$.

18. Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+2}\right)$ converges or diverges. If it converges find the sum?

19. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ if $x = \frac{2u}{u^2+v^2}$ and $y = \frac{-2v}{u^2+v^2}$.

20. What are the various elementary row operations?

21. When do we say that a matrix is in row reduced echelon form?

22. What are eigen values and eigen vectors associated with a matrix?

Section III

Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. Express $\sin^4\theta\cos^2\theta$ in terms of cosines of multiples of θ.

24. Separate $\tan^{-1}(\alpha + i\beta)$ into real and imaginary parts.

25. Locate all relative extrema and saddle points of the function $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.

26. A particle is moving along the curve whose equation is $\frac{xy^3}{1+y^2} = \frac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units per second when the particle is at $(1,2)$

    (a) At what rate is the y-coordinate of the point changing at that instant?

    (b) Is the particle rising or falling at that instant?

27. Find the Taylor series expansion for $\tan^{-1}x$ about $x = 1$.

28. Find the absolute maximum and absolute minimum of $f(x,y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices $(0,0), (3,0), (0,5)$.

29. Use the method of Lagrangian multipliers to find the maximum and minimum values of the function $f(x,y) = xy$ subject to the constraint $4x^2 + 8y^2 = 16$.

30. Solve the system of equations

$$\begin{aligned} x_1 - 7x_2 + 6x_4 &= 5 \\ x_3 - 2x_4 &= -3 \\ -x_1 + 7x_2 - 4x_3 + 2x_4 &= 7 \end{aligned}$$

31. Find the row reduced echelon form of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \\ 4 & 0 & 7 \end{pmatrix}$

    and determine its rank.

Section IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (a) Express $\cos 6\theta$ in terms of powers of $\cos\theta$.

    (b) If $\cos(x + iy) = \cos\theta + i\sin\theta$, show that $\cos 2x + \cosh 2y = 2$.

33. (a) Determine the dimensions of a rectangular box open at the top, having a volume 32 cubic feet and requiring the least material for its construction.

    (b) Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.

34. (a) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace's equation.

    (b) Given $z = 8x^2y - 2x + 3y$, $x = uv$ and $y = u - v$, use the chain rule to find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

    (c) Let f be a differentiable function of one variable and let $z = f(x^2 + y^2)$. Show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$.

35. (a) Find the general solution of the system

$$\begin{aligned} 2x_1 - 3x_2 &= 1 \\ -x_1 + 3x_2 &= 0 \\ x_1 - 4x_2 &= 3 \end{aligned}$$

    or show that the system has no solution.

    (b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ -4 & -4 & 5 \end{pmatrix}$