UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme in Statistics
Semester IV
MM 1431.4 Mathematics – IV
(Linear Algebra)

Time: 3 hours **Maximum Marks: 80**

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Are the following matrices row equivalent:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Check whether the matrix $\begin{bmatrix} 1 & 1 & -3 \\ 2 & 16 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ is singular.
3. Find the inverse of the matrix: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
4. Find the rank of the matrix: $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5. Define a dilation from $\mathbb{R}^2$ to $\mathbb{R}^2$.
6. Write down the standard matrix corresponding to the transformation of reflection in the line $x_2 = -x_1$.
7. State true or false: If A is an $m \times n$ matrix and the transformation $x \to Ax$ is onto, then $rank\, A = m$.
8. Let A be a 7×5 matrix. What must m and n be in order to define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(x) = Ax$
9. If a vector space V has a basis of n vectors, then every basis of V must consist of exactly ….. vectors.
10. Find the matrix of the quadratic form: $Q(x) = {x_3}^2 - 4x_1x_2 + 4x_1x_3$

Section-II

Answer any 8 questions from among the questions 11 to 22.

These questions carry 2 marks each.

11. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find A^n
12. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, show that $A^2 - 4A + 5I = 0$
13. Find the Eigen values of the matrix: $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

14. By reducing it to the echelon form, find the rank of the matrix: $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

15. Find the index of the nilpotent matrix: $\begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & -2 & 0 \end{bmatrix}$

16. Show that a matrix and its transpose have the same Eigen values.

17. Is $(1,-2)$ and $(-2,4)$ a basis for $\mathbb{R}^2$ over $\mathbb{R}$?

18. Define a linear transformation and check whether the transformation T is linear if T is defined by: $T(x_1,\ x_2) = (2x_1 - 3x_2,\ x_1 + 4,\ 5x_2)$.

19. Let T be the linear transformation defined by $T(e_1) = (1,4)$, $T(e_2) = (-2,9)$ and $T(e_3) = (3,-8)$, where e_1, e_2 and e_3 are columns of the 3×3 identity matrix. Check whether T is one-one or not.

20. Let $v_1 = [3 \quad 6 \quad 2]^{\mathrm{T}}$, $v_2 = [-1 \quad 0 \quad 1]^{\mathrm{T}}$, $\mathrm{x} = [3 \quad 12 \quad 7]^{\mathrm{T}}$ and $B = \{v_1, v_2\}$. Find the co-ordinate vector $[\mathrm{x}]_B$ of x relative to B.

21. Find the dimension of the null space and the column space of:

$$\begin{pmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

22. If $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, show that $A^3 - 6A^2 + 11A - 6I = 0$

Section-III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. Test for consistency and solve:

$$5x + 3y + 7z = 4; \quad 3x + 26y + 2z = 9; \quad 7x + 2y + 10z = 5$$

24. Let A be an $n \times n$ matrix with n distinct Eigen values. Prove that the corresponding Eigen vectors are linearly independent.

25. Find the Eigen vectors of the matrix: $\begin{bmatrix} 3 & -5 & -4 \\ -5 & -6 & -5 \\ -4 & -5 & 3 \end{bmatrix}$

26. Diagonalise the matrix: $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

27. Check whether {(−1, 1, 2), (2, −3, 1), (10, −14, 0)} is a basis for $\mathbb{R}^3$ over $\mathbb{R}$ or not.

28. Let $T(x_1,\ x_2) = (3x_1 + x_2,\ 5x_1 + 7x_2,\ x_1 + 3x_2)$. Show that T is one-one. Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

29. Let $b_1 = [1 \quad -3]^T$, $b_2 = [-2 \quad 4]^T$, $c_1 = [-7 \quad 9]^T$, $c_2 = [-5 \quad 7]^T$. Consider the bases of $\mathbb{R}^2$ given by $B_1 = \{b_1, b_2\}$ and $B_2 = \{c_1, c_2\}$. Find the change of co-ordinate matrix from B_2 to B_1 and the change of co-ordinate matrix from B_1 to B_2.

30. Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ and $B = \{b_1, b_2\}$; for $b_1 = [1 \quad 1]^T$, $b_2 = [5 \quad 4]^T$. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$. Show that b_1 is an Eigen vector of A. Is A diagonalizable?

31. Make a change of variable $x = Py$, that transforms the quadratic form ${x_1}^2 + 10x_1x_2 + {x_2}^2$ in to a quadratic form without cross product term. Give P and the new quadratic form.

Section-IV
Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. a. Let $T: \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation and let $\{v_1, v_2, v_3\}$ be a linearly independent set in $\mathbb{R}^n$. Show that the set $\{T\,(v_1), T\,(v_2), T\,(v_3)\}$ is also linearly independent

    b. Find four bases for $\mathbb{R}^3$ over $\mathbb{R}$, no two of which have a vector in common.

33. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$ where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. Find a basis B for $\mathbb{R}^2$ with the property that $[T]_B$ is diagonal.

34. Prove that $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable and find the diagonal form.

35. Prove that the vectors $x_1 = (2 \quad -1 \quad 4)$, $x_2 = (4 \quad 0 \quad 12)$ and $x_3 = (0 \quad 1 \quad 2)$ are linearly dependent and find a relationship between them.
