UNIVERSITY OF KERALA

Model Question Paper- M. Sc. Examination

Branch : Mathematics

MM 214 - Topology I

(2020 Admission onwards)

Time: 3 hours — Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8

Each question carries 3 marks

1. Is the limit of a convergent sequence in a metric space unique? Justify your answer.
2. Let R be the subset of $\mathbb{R}^n$ consisting of all points having only rational co-ordinates. Prove that $\bar{R} = \mathbb{R}^n$
3. Define isometric spaces. Give an example
4. Is the limit of a convergent sequence in a topological space unique? Justify your answer.
5. For $X = \{a, b\}$ with trivial topology and $A = \{a\}$, find $int\ A$, $bdy\ A$ and A'
6. For a connected space X and a continuous onto function $f : X \to Y$, prove that Y is connected.
7. Give example of two disconnected sets whose union is connected.
8. Show that every compact space has the Bolzano-Weierstrass property . $5 \times 3 = 15$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9. A. a. Define the Max Metric on $\mathbb{R}^n$. Show that it is a metric on $\mathbb{R}^n$
   b. Let (X, d) be a metric space and A subset of X. Prove that, a point $x \in X$ is a limit point of A if and only if there is a sequence of distinct points of A which converges to x.
   c. Prove that singleton sets are open in a discrete metric space.

OR

   B. a. Prove that a finite subset of a metric space has no limit points.
   b. For a subset A of a metric space X, show that $\bar{A}$ is a closed set and is a subset of every closed set containing A.
   c. Give example of two subsets A and B of a metric space X with $A \subset B$ but $bdy\ A \not\subset bdy\ B$

10. A. a. For a function $f : X \to Y$, where X and Y are metric spaces, prove that f is continuous if and only if for each open set O in Y, $f^{-1}(O)$ is open in X
   b. State and prove Baire Category Theorem

OR

B. Show that every metric space (X, d) has a unique completion

11. A. a. Let X be a topological space, $A \subset X$ and x a limit point of A. Will there always be a sequence of distinct points in A that converges to x? Justify your answer.

    b. Show that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ for any subsets A, B of a space X

    c. For a subset A of a topological space X, prove that A is open if and only if *bdy* $A \subset (X \backslash A)$

OR

B. a. Prove that a separable metric space is second countable.

   b. Prove that the property of being a Hausdorff space is both a topological property and hereditary property.

12. A. a. Prove that a topological space X is disconnected if and only if there is a continuous function from X onto a discrete two point space.

    b. Show that the connected subsets of $\mathbb{R}$ are precisely the intervals

OR

B. a. Prove that every closed and bounded interval has the fixed-point property

   b. Prove that every open, connected subset of $\mathbb{R}^n$ is path connected

13. A. a. State and prove Cantor's theorem of Deduction. Show that the requirement that the subsets E_n are bounded is necessary.

    b. Show that a continuous function from a compact metric space to an arbitrary metric space is uniformly continous

OR

B. a. State and prove the Lindelof theorem

   b. Show that, every open cover of a compact metric space X has a Lebesgue number

   c. For the one point compactification $(X_\infty, \mathcal{T}_\infty)$ of $(X, \mathcal{T})$, show that if X_∞ is Hausdorff, then X is both Hausdorff and locally compact

. $5 \times 12 = 60$