Model Question Paper

First Semester MSc Degree Examination

Statistics with Specialization in Data Analytics

(2020 Admission onwards)

STSD212:Probability Theory

Time : 3 hours Max. Marks : 75

Part A

Answer any **five** questions. Each carries 3 marks

1. Define sigma ring with an example.
2. If f and g are measurable functions from a non empty set Ω to R^* and $k \in R$, the show that f+ g and max(f, g) are measurable functions.
3. Define expectation of a simple random variable.
4. Define probability distribution function of a random variable.
5. Define discrete random variable with example.
6. Define almost sure convergence of a sequence of random variables. Illustrate the concept through an example.
7. State Bochners theorem.
8. State Kolmogorov's strong law of large numbers.
9. State Bernoulli's WLLN.
10. State Chebychev's inequality.

Part B

Answer any **three** questions. Each carries 12marks

11. State and prove Borel- Cantelli lemma.
12. a) Explain additive and sigma additive set function with example.
    b) Give an example of a set function which is finitely additive but not sigma additive.
    c) Explain sigma-finite set function with suitable example.
13. Establish the concept of independence of a sequence of random variables.
14. Establish the properties distribution function $F_X(x)$ of a random variable X.
15. State inversion theorem. Use it to reduce Fourier inversion theorem. Find the characteristic function of Cauchy distribution.

16. If X is a non negative random variable with distribution function F, then prove that

$$E(X)=\int_0^{\infty}[1-F(x)]\,dx$$
.

Part C

Answer any **two** questions. Each carries 12 marks

17. State and prove Lyapunov inequality.
18. Prove that convergence in probability implies convergence in distribution.
19. Show that Lyapunov condition for central limit theorem implies Lindberg condition for central limit theorem.
20. Let $\{X_n\}$be a sequence of independent random variables with $P\{X_n=2^n\}=P\{X_n=-2^n\}=\frac{1}{2}$. Check whether WLLN hold for the sequence of random variables.